

OBSERVATIONS AND ANALYSIS OF THE FIELD CONTACT BINARY V728 HERCULIS<sup>1</sup>

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## ABSTRACT

The W UMa system V728 Her has been observed photometrically in  $B$ ,  $V$ , and  $I_C$  passbands at the RAO over five seasons and spectroscopically at DAO over three seasons to obtain radial velocities. New times of minima are provided and a period analysis confirms previous ephemerides. The light and radial velocity curves were analyzed with the latest University of Calgary enhancements to the Wilson–Devinney program and new solutions found. The best model is that involving a contact system with convective atmospheres. In probing the best solution possible, use was made of the enhanced reflection and second-order limb-darkening calculations available in the 1993 version of the WD program. The mass ratio is found to be  $0.1786 \pm 0.0023$ . The contact parameter is found to be  $f = 0.71 \pm 0.11$ . The masses are determined to be  $1.654 \pm 0.037 M_\odot$  and  $0.295 \pm 0.009 M_\odot$  and the radii are  $1.784 \pm 0.015 R_\odot$  and  $0.867 \pm 0.054 R_\odot$ , respectively. The determined temperature difference,  $T_2 - T_1$  is  $165 \pm 19$  K, and the luminosities for components 1 and 2 are  $5.5 \pm 0.5 L_\odot$  and  $1.4 \pm 0.3 L_\odot$ , respectively. © 1995 American Astronomical Society.

## 1. INTRODUCTION

V728 Her (SVS 2086) has been found to be located at  $\alpha = 17^h 18^m 05^s$ ,  $\delta = 41^\circ 50' 41''$  (2000) by Agerer *et al.* (1988). Its variability was discovered by Kurochkin (1977) in a photographic survey of stars in the region of M92, but its position was misreported, resulting in an incorrect position in the *General Catalogue of Variable Stars* (Kholopov 1985). However, Kurochkin (1977) correctly identified the variable on his chart.

Kurochkin classified V728 Her as type EW and derived a period of 0.446 250 d on the basis of a photographic light curve which displayed considerable scatter. Ciardo *et al.* (1985) obtained partial light curves in  $U$ ,  $B$ , and  $V$  and determined a time of minimum. Faulkner (1986) obtained a time of minimum, noting that it did not fit Kurochkin's ephemeris. Nelson *et al.* (1988) reported  $B$ ,  $V$ , and  $I_C$  light curves obtained in 1986 and 1987, determined three times of

minimum and a preliminary period of 0.471 302 d. They also obtained spectra and determined the spectral type to be  $F3 \pm 1$ . Agerer *et al.* (1988) obtained full  $B$  and  $V$  curves and obtained new elements. Samec & Butcher (1989) presented three times of minima, full  $B$  and  $V$  light curves, and similar elements to those of Agerer *et al.* (1988). Other timings were presented by Hübscher *et al.* (1989) and in BBSAG (1988a, b). Finally, Samec (1990) performed WD modeling (Wilson & Devinney 1971) based on their  $B$  and  $V$  light curves alone.

At the University of Calgary's Rothney Astrophysical Observatory, full light curves in  $B$ ,  $V$ , and  $I_C$  have been obtained over several seasons, resulting in eight new times of minima. Analysis essentially confirms the ephemerides of Agerer *et al.* (1988) and Samec & Butcher (1989). Radial velocity curves have been obtained, and Wilson–Devinney modeling has been completed and system parameters have been determined.

## 2. OBSERVATIONS

Observations were carried out between April and July in each of the years 1986–90 with the Rapid Alternate Detec-

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TABLE 1. V728 photoelectric observations.\*

HJD	Phase	B	V	$I_c$	HJD	Phase	B	V	$I_c$
2440000+					2440000+				
6591.9000	0.50819	...	1.722	2.140	6591.9047	0.51831	...	1.724	2.121
6591.9107	0.53100	...	1.705	2.136	6591.9176	0.54558	1.125	1.695	2.090
6595.8310	0.84928	0.932	1.442	...	6595.8393	0.86687	0.984	1.497	1.925
6595.8462	0.88156	0.989	1.511	1.940	6595.8537	0.89738	1.045	1.571	1.926
6595.8606	0.91215	1.098	1.599	2.023	6595.8673	0.92628	1.123	1.587	2.052
6595.8744	0.94141	1.140	1.671	2.045	6595.8809	0.95512	1.194	1.678	2.115
6595.8952	0.98553	1.227	1.712	2.135	6595.9026	0.00127	1.215	1.709	...
6612.7582	0.76623	...	1.336	1.786	6612.7659	0.78261	0.821	1.354	1.783
6612.7803	0.81314	0.880	1.339	1.789	6612.7882	0.83005	0.878	1.382	1.830
6612.7955	0.84546	0.871	1.417	1.854	6612.8035	0.86237	0.928	1.445	1.864
6612.8114	0.87928	0.968	1.460	1.900	6612.8198	0.89702	1.003	1.522	1.933
6612.8277	0.91380	1.062	1.543	1.968	6612.8354	0.93003	1.074	1.595	1.995
6612.8424	0.94501	1.141	1.623	2.041	6612.8501	0.96125	1.145	1.661	2.058
6612.8582	0.97843	1.185	1.679	2.099	6612.8728	0.00937	1.200	1.683	2.101
6612.8792	0.02314	1.192	1.681	2.079	6612.8871	0.03990	1.183	1.653	2.065
6612.8936	0.05367	1.147	1.601	2.046	6612.9013	0.07003	...	1.591	...
6612.9078	0.08378	1.074	1.549	1.976	6613.7610	0.89402	...	1.475	1.919
6613.7676	0.90817	0.966	1.517	1.965	6613.7753	0.92438	1.053	1.529	...
6613.7860	0.94707	1.103	1.618	2.015	6613.7941	0.96432	1.156	1.650	2.058
6613.8020	0.98112	1.172	1.657	2.072	6613.8101	0.99835	1.139	1.687	2.111
6613.8169	0.01280	1.177	1.668	2.089	6613.8249	0.02959	1.142	1.631	2.062
6613.8317	0.04404	1.095	1.611	2.036	6613.8400	0.06171	1.045	1.574	1.994
6613.8468	0.07616	1.040	1.544	1.950	6613.8546	0.09280	0.990	1.483	1.921
6613.8615	0.10725	0.944	1.468	1.894	6613.8695	0.12435	0.912	1.425	1.871
6613.8763	0.13878	0.892	1.433	1.851	6613.8842	0.15543	0.875	1.372	1.831
6613.8910	0.16986	0.854	1.395	1.808	6613.8988	0.18652	0.847	1.345	1.776
6613.9056	0.20097	0.832	1.300	1.736	6614.7283	0.94655	...	1.586	2.006
6614.7341	0.95894	...	1.587	2.033	6614.7411	0.97367	1.127	1.701	2.091
6614.7531	0.99915	1.178	1.682	2.080	6614.7606	0.01507	1.179	1.671	2.081
6614.7686	0.03202	1.119	1.655	2.060	6614.7754	0.04647	1.109	1.618	2.057
6614.7832	0.06296	1.172	1.559	1.992	6614.7900	0.07741	1.016	1.548	1.960
6614.7989	0.09627	0.955	1.495	1.885	6614.8045	0.10832	0.906	1.453	1.897
6614.8145	0.12941	0.917	1.431	1.868	6614.8213	0.14386	0.870	1.392	1.836
6614.8297	0.16169	0.849	1.376	1.831	6614.8365	0.17614	0.837	1.375	1.824

\*Table 1 is presented in its complete form in the AAS CD-ROM series, Volume 5, 1995. The first page of this table is presented here for guidance regarding its form and content.

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tion System (RADS) on the 41 cm Cassegrain telescope at the Rothney Astrophysical Observatory of the University of Calgary (Milone *et al.* 1982; Milone & Robb 1983). This is a four channel system involving a gated chopping secondary with full control of the steps, azimuthal angle, and timing so that the variable, comparison, and two sky channels may be alternately observed over intervals as short as 0.1 s and total integration times as long as 120 s. Therefore, fluctuations in sky brightness and transparency can be compensated for as long as the fluctuations occur at greatly different frequencies from the sampling frequency.

Star "a" on the finder chart of Kurochkin (1977) was used as a comparison [ $\alpha=17^h 19^m 19^s$ ,  $\delta=41^\circ 56' 39''$  (2000)]; the check star was SAO 46620 [BD+42°2822,  $\alpha=17^h 18^m 19^s$ ,  $\delta=42^\circ 06' 39''$  (2000)]. The comparison star is not as well matched to the variable in color as we would like:  $\Delta(B-V) \approx 0.45$ , a consequence of the chopping limit of the RADS photometry (see below). Table 1 lists a total of 92, 154, 200, 119, and 131 differential  $V$  magnitudes obtained for the five years 1986–90, respectively, with approximately the same number of  $B$  and  $I_c$  values over the same interval. Atmospheric extinction coefficients were obtained using values determined or estimated nightly from the comparison star magnitudes, when the data were spread over a sufficiently wide range of air mass. Transformation coefficients were obtained from the standards of Landolt (1973) via a differential form of the Hardie (1962) method. No zero points were determined. From nine differential observations of the check star; the mean standard errors (mse's) of single observations of the check–comparison star photometry are:  $\pm 0.011$  ( $B$ ),

TABLE 2. Log of spectroscopic observations.

Plate Number	HJD	Phase	$V_1$	$V_2$	Std <sup>a</sup>
	2440000+		km/s		
62085	6649.7352	0.2292	-16.12 ±4.55	301.03 ±14.32	2
62085	6649.7352	0.2292	-9.22 4.91	292.27 13.81	1
62087	6649.7414	0.2424	-46.85 5.40	296.38 18.78	2
62087	6649.7414	0.2424	-36.51 5.27	259.62 14.10	1
62098	6649.8266	0.4232	9.72 3.31	254.30 14.30	2
62098	6649.8266	0.4232	11.64 3.06	240.33 8.53	1
62100	6649.8331	0.4370	20.45 2.51	210.92 11.23	2
62100	6649.8331	0.4370	22.12 2.10	210.25 8.09	1
62160	6650.7452	0.3723	0.36 3.39	248.92 10.65	2
62160	6650.7452	0.3723	5.25 5.39	241.35 10.75	1
62162	6650.7513	0.3852	-3.61 3.11	226.96 7.82	2
62162	6650.7513	0.3852	-0.61 3.47	230.15 5.94	1
62164	6650.7567	0.3967	4.79 3.04	224.58 6.88	2
62164	6650.7567	0.3967	6.26 2.79	226.30 5.33	1
62166	6650.7628	0.4096	9.71 3.63	218.59 10.52	2
62166	6650.7628	0.4096	16.60 3.30	215.06 7.02	1
62239	6651.8851	0.7910	97.43 2.89	-198.75 4.55	2
62239	6651.8851	0.7910	102.96 4.23	-210.71 6.56	1
71019	6970.9101	0.7146	50.42 4.17	-253.86 13.02	1
71019	6970.9101	0.7146	62.18 3.62	-242.57 10.61	2
71019	6970.9101	0.7146	80.64 4.22	-218.49 12.07	2
71021	6970.9157	0.7265	67.49 6.36	-225.92 24.53	1
71021	6970.9157	0.7265	72.25 4.71	-227.06 19.35	2
71021	6970.9157	0.7265	94.02 5.71	-213.91 15.09	2
71023	6970.9213	0.7383	60.75 10.86	-263.50 26.40	1
71023	6970.9213	0.7383	61.16 14.81	-261.11 51.75	2
71023	6970.9213	0.7383	90.45 20.36	-241.26 14.50	2
71082	6971.8412	0.6902	68.52 3.70	-239.55 11.46	1
71082	6971.8412	0.6902	80.57 3.52	-230.32 9.56	2
71082	6971.8412	0.6902	94.38 3.72	-211.93 8.92	2
71084	6971.8488	0.7064	58.86 2.34	-200.48 7.05	1
71084	6971.8488	0.7064	64.75 2.92	-219.81 8.43	2
71084	6971.8488	0.7064	80.22 3.92	-189.02 11.35	2
71086	6971.8549	0.7193	63.73 4.12	-227.88 10.23	1
71086	6971.8549	0.7193	76.64 4.27	-235.91 8.45	2
71086	6971.8549	0.7193	84.71 4.11	-207.10 7.55	2
71101	6971.9069	0.8296	59.57 3.07	-185.36 7.50	1
71101	6971.9069	0.8296	74.87 2.12	-191.29 6.62	2
71101	6971.9069	0.8296	88.56 ±4.54	-159.41 ±10.33	2
71104	6971.9152	0.8472	65.98 4.33	-196.18 9.10	1
71104	6971.9152	0.8472	72.90 4.20	-182.52 7.64	2
71104	6971.9152	0.8472	92.61 4.85	-173.94 8.63	2
71106	6971.9220	0.8617	56.75 2.57	-179.73 4.85	1
71106	6971.9220	0.8617	65.35 1.85	-174.37 4.06	2
71106	6971.9220	0.8617	80.83 2.86	-145.84 4.93	2
72387	7041.7063	0.9336	21.52 4.10	-61.77 5.03 <sup>b</sup>	1
72387	7041.7063	0.9336	22.23 3.48 <sup>b</sup>	...	2
72391	7041.7618	0.0513	16.24 1.57	...	1
72391	7041.7618	0.0513	26.11 0.59	...	1
72439	7042.6650	0.9678	12.77 0.92 <sup>b</sup>	...	2
72439	7042.6650	0.9678	22.43 1.27 <sup>b</sup>	...	1
72563	7043.8010	0.3782	-0.14 2.72	224.07 5.06	2
72563	7043.8010	0.3782	14.54 1.99	219.27 4.51	1
72565	7043.8149	0.4077	2.81 3.27	221.03 7.07	2
72565	7043.8149	0.4077	18.43 2.44	220.10 6.63	1
72567	7043.8307	0.4412	1.60 1.72	226.28 14.34	2
72567	7043.8307	0.4412	8.35 2.10	216.23 11.55	1
72709	7045.6861	0.3781	-5.12 2.14	260.76 7.51	2
72709	7045.6861	0.3781	11.07 2.74	252.92 8.96	1
72711	7045.6998	0.4072	-1.82 2.37	241.03 7.53	2
72711	7045.6998	0.4072	13.08 2.75	240.42 8.96	1
72713	7045.7135	0.4363	20.75 1.59	...	2
72713	7045.7135	0.4363	23.88 3.11	230.32 11.53	1
88890	7283.0014	0.9259	-18.39 1.97 <sup>b</sup>	...	2
81357	7316.7564	0.5490	-10.74 0.24 <sup>b</sup>	...	2
81359	7316.7740	0.5863	-7.68 1.32 <sup>b</sup>	...	2
81361	7316.7872	0.6143	-7.09 0.13 <sup>b</sup>	...	2
81396	7317.9017	0.9791	75.31 6.64	-100.40 14.64 <sup>b</sup>	2

<sup>a</sup>Radial velocity standards: 1 = HD 154417, 2 = HD 181096

<sup>b</sup>Because of the proximity to the crossing points (and resulting ambiguity), these radial velocities were not used in the modeling.

$\pm 0.010$  ( $V$ ), and  $\pm 0.013$  ( $I_c$ ). There is no evidence of variation. The phasing of the data is discussed below.

Intensified reticon spectra were obtained with the 21121 spectrograph on the 1.8 m telescope at the Dominion Astrophysical Observatory (DAO) in each of 1986–88. The nominal dispersion of this instrument was 15 Å/mm. A log of the radial velocity observations and determinations is presented in Table 2. The observations were reduced with the REDUCE

TABLE 3. O–C data.

Minimum 2400000+	$\Delta t/P$	cycle	O–C	Reference	$Q^a$
45882.3750	-2264.9945	-2265	0.00256	Ciardo <i>et al.</i> 1985	1
46257.7518	-1468.5011	-1468.5	-0.00058	Faulkner 1986	1
46612.8677	-714.9984	-715.	0.00072	Nelson <i>et al.</i> 1988	1
46613.8097	-712.9996	-713.	0.00014	Nelson <i>et al.</i> 1988	1
46949.8348	0.0047	0.	-0.00224	Nelson <i>et al.</i> 1988	1
46990.8388	86.9997	87.	-0.00015	present paper	1
47004.7431	116.5025	116.5	0.00114	present paper	1
47304.4690	752.4760	752.5	-0.01136	('Diethelm, BBSAG 88') <sup>b</sup>	0
47309.8990	763.9976	764.	-0.00110	present paper	1
47322.3914	791.0045	791.	0.00214	BAV 1989	0
47322.3928	791.0075	791.	0.00354	BAV 1989	0
47322.3914	791.0045	791.	0.00214	BAV 1989	0
47322.3928	791.0075	791.	0.00354	BAV 1989	0
47323.8030	793.4998	793.5	-0.00012	Samec 1990	1
47323.8031	793.5000	793.5	0.00002	present paper	1
47326.8656	799.9993	800.	-0.00088	Samec 1990	1
47327.8087	801.9993	802.	-0.00236	present paper	1
47328.7507	803.9981	804.	-0.00093	Samec 1990	1
47353.4937	856.4991	856.5	-0.00049	Agerer <i>et al.</i> 1988	1
47365.5124	882.0010	882.	0.00080	Agerer <i>et al.</i> 1988	1
47366.4551	884.0012	884.	0.00193	Agerer <i>et al.</i> 1988	1
47378.4730	909.5014	909.5	0.00091	Agerer <i>et al.</i> 1988	1
47391.4490	937.5344	937.5	0.01622	BBSAG 1988a	0
47415.5450	988.6625	988.5	0.07660	BBSAG 1988a	0
47439.3150	1039.0989	1039.	0.04661	BBSAG 1988b	0
47655.8233	1497.9971	1498.	-0.00137	present paper	1
47659.8284	1506.4954	1506.5	-0.00221	present paper	1
47695.8834	1582.9986	1583	-0.00041	present paper	1

<sup>a</sup> Only data with Quality values 1 were included in present fittings.<sup>b</sup> Cited in Agerer *et al.* 1988.

and VCROSS software of Hill (1982). The comparison stars were HD 154417 and HD 181096, observed during each observing run. The separate cross-correlation results are included in Table 2.

### 3. PERIOD ANALYSIS

Agerer *et al.* (1988) analyzed all available times of minima and derived two periods, 0.471 285 2 d, based on early photographic and photoelectric data, and 0.471 286 8 d, based on photoelectric data only. Included in the first set of data were Kurochkin's (1977) photographic times of minima, which, however are not real minima but merely isolated times of low-light images on photographic plates and therefore should be assigned very low weight.

The second ephemeris from Agerer *et al.* (1988),

$$\text{JD Hel Min I} = 2\,446\,949.8370(4) + 0.471\,286\,8(4) E, \quad (1)$$

where the parentheses contain the uncertainties in units of the last decimal place, was used to calculate the cycle number for 20 selected photoelectric minima.

A total of eight new times of minima (in each of three passbands) have been obtained for the current study. All available times of minima (including those with approximate, photographic low-light dates) are listed in Table 3. Our data were phased with the following elements, determined from an earlier, unpublished analysis by RHN:

$$\text{JD Hel Min I} = 2\,446\,949.836\,72 + 0.471\,286\,6 E. \quad (2)$$

Subsequent least squares analyses of all times of minima yielded the following ephemeris:

$$\text{JD Hel Min I} = 2\,446\,949.837\,96(23)$$

$$+ 0.471\,286\,19(27) E, \quad (3)$$

where the errors are mse's. Since this analysis indicated slightly different elements, in subsequent light curve modeling we adjusted the phase shift parameter.

Differences among the elements of the three ephemerides are scarcely significant; therefore, the periods of Agerer *et al.* (1988) given above, and Samec & Butcher (1989), [viz., 0.471 286 4(2) d] and their epochs are essentially confirmed. While no evidence of significant period variation is seen if low-precision data are excluded, there is sufficient recorded light curve variation to echo Samec's call for additional observations in future to check on this possibility. In the present data set, we have only incomplete light curves from season to season, and therefore only the mean light curve was modeled. The existence of runs of residuals from season to season suggests light curve variability over intervals of the order of a few cycles.

### 4. LIGHT CURVE MODELING

The modeling was carried out with the differential corrections mode of the University of Calgary's improved version of the Wilson–Devinney program (Wilson 1979, 1990; Milone *et al.* 1992) initially using the Cyber 205 at the University of Calgary and the Digital VAX 4500 at the College of New Caledonia. When an updated WD code became available (Wilson 1992, 1993), the Kurucz atmosphere grids were incorporated into the WD code, a version referred to as WD93K93 (Milone *et al.* 1995a,b). Further modeling was done with WD93K93 on the IBM RS6000 and compute servers and on 486 and Pentium computers at the University of Calgary.

Because of the time constraints of running unbinned data, flux values were binned in phase widths of 0.01. Weights were assigned based on the number of points in each bin. The weights of the unaveraged radial velocity data reflect the inverse mean standard error of the fit of a Gaussian to the cross-correlation curve in the vicinity of the peak. The radial velocity input data, in quintets of phase, radial velocities, and weight are listed in Table 4.

The spectral type was determined to be  $F3 \pm 1$  spectral subclass (Nelson *et al.* 1988), and based on this information, initial values for both the gravity darkening exponent and albedo were taken to be 1.00, appropriate for a system with radiative atmospheres. Again, based on the spectral type, the temperature of the primary was taken to be  $6622 \pm 150$  K (Popper 1980). Because of the general appearance of the light curves, the system was assumed to be a contact binary so that the differential corrections program was run in mode 3. Preliminary modeling indicated an over-contact binary, wherein each star overfills its Roche lobe, thus verifying the assumption; this assumption was also confirmed by Samec (1990). In WD93K93,  $\log g$  values (in cgs units) were calculated using masses  $\mathcal{M}_1$  and  $\mathcal{M}_2$  derived from initial (and, later, corrected) values for the semimajor axis  $a$ , the mass ratio,  $q = \mathcal{M}_2/\mathcal{M}_1$ , and the effective stellar radii  $R_1$  and  $R_2$ . Values for the limb-darkening coefficients  $x_1$ ,  $x_2$  were taken from Van Hamme's tables (Van Hamme 1993). Initial values for the other parameters were taken from Samec (1990). The



TABLE 4. WD93K93 radial velocity input data.

$\phi$	V	W	$\phi$	V	W	$\phi$	V	W	$\phi$	V	W	$\phi$	V	W
Component 1:														
.0513	0.0162	19	.0513	0.0261	20	.2292	-.0092	06	.2292	-.0161	07	.2424	-.0468	06
.2424	-.0365	06	.3723	0.0052	06	.3723	0.0004	09	.3781	0.0111	11	.3781	-.0051	14
.3782	-.0001	11	.3782	0.0145	15	.3852	-.0006	09	.3852	-.0036	10	.3967	0.0048	10
.3967	0.0063	11	.4072	0.0131	11	.4072	-.0018	13	.4077	0.0184	12	.4077	0.0028	13
.4096	0.0097	08	.4096	0.0166	09	.4232	0.0097	09	.4232	0.0116	10	.4363	0.0239	10
.4363	0.0207	19	.4370	0.0204	12	.4370	0.0221	14	.4412	0.0083	14	.4412	0.0016	17
.6902	0.0944	08	.6902	0.0685	08	.6902	0.0806	09	.7064	0.0802	08	.7064	0.0648	10
.7064	0.0589	13	.7146	0.0806	07	.7146	0.0504	07	.7146	0.0622	08	.7193	0.0766	07
.7193	0.0637	07	.7193	0.0847	07	.7265	0.0675	05	.7265	0.0940	05	.7265	0.0723	06
.7383	0.0904	01	.7383	0.0612	02	.7383	0.0608	03	.7910	0.1030	07	.7910	0.0974	10
.8296	0.0886	07	.8296	0.0596	10	.8296	0.0749	14	.8472	0.0926	06	.8472	0.0660	07
.8472	0.0729	07	.8617	0.0808	10	.8617	0.0567	12	.8617	0.0654	16			
Component 2:														
.2292	0.3010	02	.2292	0.2923	02	.2424	0.2964	02	.2424	0.2596	02	.3723	0.2413	03
.3723	0.2489	03	.3781	0.2529	03	.3781	0.2608	04	.3782	0.2241	06	.3782	0.2193	07
.3852	0.2270	04	.3852	0.2301	05	.3967	0.2246	04	.3967	0.2263	06	.4072	0.2404	03
.4072	0.2410	04	.4077	0.2210	04	.4077	0.2201	05	.4096	0.2186	03	.4096	0.2151	04
.4232	0.2543	02	.4232	0.2403	04	.4363	0.2303	03	.4370	0.2109	03	.4370	0.2103	04
.4412	0.2263	02	.4412	0.2162	03	.6902	-.2395	03	.6902	-.2303	03	.6902	-.2119	03
.7064	-.1890	03	.7064	-.2198	04	.7064	-.2005	04	.7146	-.2539	02	.7146	-.2185	02
.7146	-.2426	03	.7193	-.2279	03	.7193	-.2359	04	.7193	-.2071	04	.7265	-.2259	01
.7265	-.2271	02	.7265	-.2139	02	.7383	-.2611	01	.7383	-.2413	01	.7383	-.2635	01
.7910	-.2107	05	.7910	-.1988	07	.8296	-.1594	03	.8296	-.1854	04	.8296	-.1913	05
.8472	-.1962	03	.8472	-.1739	03	.8472	-.1825	04	.8617	-.1458	06	.8617	-.1797	06
.8617	-.1744	07												

non-adjusted and initial parameters are shown in Table 5. The other parameters were varied as adjustments required. The limb-darkening coefficients, and the value of  $L_1$  for the radial velocity curves (assumed to have an effective wavelength of  $0.41 \mu\text{m}$ ) were altered accordingly. In the final modeling, second-order coefficients were used, again from Van Hamme's tables, and the option to apply a single reflection iteration was invoked.

Modeling iterations were made with the method of multiple subsets, of sufficient number to permit adjustment of each parameter included in the largest set with parameter correlations less than  $\sim 0.3$ . In the initial runs, nine parameters were adjusted: semimajor axis  $a$ , system radial velocity  $V_\gamma$ , inclination  $i$ , temperature  $T_2$ , potential  $\Omega_1$ , mass ratio  $q$ , luminosities  $L_1(B)$ ,  $L_1(V)$ ,  $L_1(I_C)$ , where the subscripts refer to the component ("1" is that of the star eclipsed at primary minimum, here taken at phase 0). In later runs, the phase shift was also adjusted. No spots were found necessary and trials revealed that third light was not significant. The

solution for this run converged after 106 iterations, the result of a multiple of small adjustments in a very shallow region of parameter space.

Following the binned-data differential corrections modeling on the RS-6000, the simplex program, LC93KS (Kallrath 1993), was used to test the uniqueness of the solution. The first converged solution resulted in very high values of  $T_2$  (more than 7200 K) but not to a significantly improved fit to the light curve. Since the distribution of data sometimes influences the areas of parameter space searched by the simplex algorithm (Kallrath and Linnell, 1987), we increased the weights of data within 0.10 of the minimum by 0.5 and those between 0.10 and 0.15 by 0.3, and reran LC93KS. The resulting solution was then checked in WD93K93 and the errors of the parameters ascertained. These results were somewhat improved but still not acceptable. Since the simplex-enhanced program had never before failed to provide an optimum value in light curve analysis, this failure indicated that despite careful attention to detail, we must have overlooked something. Further trials were clearly in order.

From experience gained from doing another low- $q$  system, H235 in the open cluster NGC 752 (Milone *et al.* 1995a,b), the radial velocities were separately modeled, with the phase shift parameter allowed to vary. The value of  $q$  was significantly different, although the other parameters, which were then further modeled, with the new and unadjusted value of  $q$ , were not significantly different from the earlier combined trials. In the end, neither light nor radial velocity curves were particularly well fit, so yet another level of analysis was carried out.

First the unbinned data were used but the resulting improvement was not significant. Then the enhanced reflection mode and second-order limb-darkening features of the 1993 WD program, also present in WD93K93, were invoked. This

TABLE 5. Unadjusted and initial modeling parameters.

Parameter (unit)	Star 1	Unadjusted/Initial Value	Star 2
$a$ ( $R_\odot$ )		3.435	
$e$		0	
$F$	1		1
$g$	1.00* (0.32)		1.00* (0.32)
$A$	1.00* (1.00)		1.00* (1.00)
$x_{bol}$	0.477* (0.643, 0.240)		0.476* (0.638, 0.248)
$x_B$	0.636* (0.804, 0.247)		0.636* (0.796, 0.256)
$x_V$	0.529* (0.712, 0.281)		0.529* (0.698, 0.282)
$x_I$	0.363* (0.557, 0.272)		0.363* (0.539, 0.281)
$\Delta\phi$		0	
$T$ (K)	6622		6622
$\Omega$	2.275		2.275
$q = M_2/M_1$		0.2484	

\*These are for the radiative trials; the convective trial data are in parentheses, pairs of limb-darkening values indicate second order limb-darkening coefficients, defined by Wilson (1993). Coefficients were periodically reinterpolated from Van Hamme's (1994) tables whenever  $\log g$  and  $T_2$  adjustments required it.

TABLE 6. WD93K93 modeling results.

Parameter	Radiative Models		Convective Models	
	Binned	Unbinned	Binned	Unbinned
$a$ (R <sub>*</sub> )	3.435(25) <sup>a</sup>	3.292(19)	3.197(23)	3.180(16)
$e^b$	0	0	0	0
$F_1 = F_2^b$	1	1	1	1
$g_1 = g_2^b$	1	1	0.32	0.32
$A_1 = A_2^b$	1	1	0.50	0.50
$x_1, y_1$ (bol)	0.479, 0.000	0.640, 0.241	0.640, 0.243	0.640, 0.243
$x_2, y_2$ (bol)	0.478, 0.000	0.641, 0.246	0.638, 0.248	0.638, 0.248
$x_1, y_1$ (B)	0.636, 0.000	0.804, 0.240	0.804, 0.247	0.804, 0.247
$x_2, y_2$ (B)	0.635, 0.000	0.794, 0.262	0.796, 0.256	0.796, 0.256
$x_1, y_1$ (V)	0.519, 0.000	0.709, 0.278	0.712, 0.281	0.712, 0.281
$x_2, y_2$ (V)	0.517, 0.000	0.698, 0.284	0.698, 0.282	0.698, 0.282
$x_1, y_1$ (I)	0.363, 0.000	0.552, 0.275	0.557, 0.272	0.557, 0.272
$x_2, y_2$ (I)	0.361, 0.000	0.541, 0.277	0.539, 0.281	0.539, 0.281
$\Delta\phi$	0.0000(5)	0.0036(3)	0.0024(4)	0.0032(3)
$V_r$ (km/s)	34.8(8)	35.3(4)	37.3(6)	37.3(4)
$i$ (deg)	66.20(61)	65.21(42)	68.14(60)	69.20(37)
$T_1$ (K)	6622 <sup>b</sup>	6622 <sup>b</sup>	6622 <sup>b</sup>	6622 <sup>b</sup>
$\Delta T = T_2 - T_1$ (K)	+22(38)	+278(43)	+154(36)	+165(19)
$T_2$ (K)	6640(155) <sup>c</sup>	6898(156) <sup>c</sup>	6776(154) <sup>c</sup>	6787(151) <sup>c</sup>
$\Omega_1 = \Omega_2$	2.275(10)	2.097(7)	2.085(7)	2.097(5)
$q = M_2/M_1$	0.2484(37)	0.1807(21)	0.1780(24)	0.1786(23)
$L_1, B$ (L <sub>*</sub> )	9.045(97)	9.360(120)	9.371(107)	9.572(61)
$L_1, V$ (L <sub>*</sub> )	9.082(85)	9.416(97)	9.498(95)	9.614(54)
$L_1, I$ (L <sub>*</sub> )	9.088(70)	9.523(71)	9.578(79)	9.699(48)
$R_1$ (pole) (a)	0.4873(17)	0.5165(13)	0.5191(14)	0.5160(13)
$R_1$ (side) (a)	0.5321(24)	0.5727(20)	0.5765(21)	0.5717(19)
$R_1$ (back) (a)	0.5619(28)	0.6021(22)	0.6063(24)	0.6004(21)
$R_2$ (pole) (a)	0.2662(61)	0.2520(56)	0.2528(64)	0.2495(60)
$R_2$ (side) (a)	0.2798(76)	0.2663(71)	0.2675(81)	0.2633(76)
$R_2$ (back) (a)	0.3321(181)	0.3388(263)	0.3466(346)	0.3314(258)
$\sum w r_i^2$	0.089974	0.153873	0.060475	0.145517
$\sigma_1$	0.0145	0.0193	0.0125	0.0188

<sup>a</sup>All errors cited are probable errors in units of the last specified place.<sup>b</sup>assumed and unadjusted<sup>c</sup>The errors in  $T_2$  are computed from the assumed error in  $T_1$ , 150 K, and that found for  $\Delta T$ .

improved the results, but not greatly. Finally, the limb darkening and albedos appropriate for convective atmospheres were assumed. The early spectral type would seem to preclude this type of model, but it seemed worth trying, since it is known that chromospheric emission features characteristic of stars with active chromospheres are sometimes seen in the spectra of early F stars when strong magnetic fields are present. The attempt is also justified by the slight O'Connell effect noted by Samec (1990), who assumed convective atmospheres for his own modeling. The resulting fits were improved significantly. The radiative and convective solutions for both binned and unbinned data sets are summarized in Table 6. Following these trials, the binned data were remodeled with the convective and other parameters which had been used for the unbinned data, and with those initial parameters. Full convergence was achieved after only a few iterations. These results are tabulated in the next to last column of Table 6, where the figures of merit are the sums of the squares of the weighted residuals and the mse of a single observation of average weight. The simplex code was used to test the uniqueness of the solution. The converged solution also gave a higher  $T_2$  than WD93K93 modeling had revealed, but not by a significant amount, 6814 K compared to 6787 K, with an uncertainty in  $\Delta T \sim 26$  K (mse). These results are summarized in Table 7; the errors in the parameters are from a DC run with WD93K93 performed with the simplex parameters. In addition, results of a WD93K93 run, started with the LC93KS parameters and iterated to full convergence, are also shown. The latter results are not significantly different from the previous WD93K93 results.

Theoretical radial velocity curves and synthetic light curves for the final model were generated using the LC subprogram of WD93K93 and are plotted along with the observed data in Figs. 1 and 2. The observed data in Fig. 2 have been

TABLE 7. Simplex (unbinned data) modeling results.

Parameter	Simplex (unbinned data) modeling results.	
	Best LC93KS Model	Adjusted WD93K93 Model
$a$ (R <sub>*</sub> )	3.214(17) <sup>a</sup>	3.189(15)
$e^b$	0	0
$F_1 = F_2^b$	1	1
$g_1 = g_2^b$	0.32	0.32
$A_1 = A_2^b$	0.50	0.50
$\Delta\phi$	0.0022(3)	0.0032(3)
$V_r$ (km/s)	37.4(4)	37.2(4)
$i$ (deg)	68.24(34)	68.65(37)
$T_1$ (K)	6622 <sup>b</sup>	6622 <sup>b</sup>
$\Delta T = T_2 - T_1$ (K)	+192(18)	+172(23)
$T_2$ (K)	6814(151) <sup>c</sup>	6794(152) <sup>c</sup>
$\Omega_1 = \Omega_2$	2.1055(63)	2.0966(56)
$q = M_2/M_1$	0.1805(22)	0.1805(21)
$L_1$ (L <sub>*</sub> )	9.414(59)	9.506(68)
$L_1$ (L <sub>*</sub> )	9.523(55)	9.552(60)
$L_1$ (L <sub>*</sub> )	9.576(45)	9.640(50)
$R_1$ (pole) (a)	0.5142(12)	0.5165(11)
$R_1$ (side)	0.5690(18)	0.5726(16)
$R_1$ (back)	0.5974(20)	0.6019(18)
$R_2$ (pole)	0.2489(56)	0.2517(53)
$R_2$ (side)	0.2624(71)	0.2660(67)
$R_2$ (back)	0.3270(225)	0.3381(245)
$\sum w r_i^2$	0.152928	0.145951
$\sigma_1$	0.0192	0.0188

<sup>a</sup>All errors cited are probable errors in units of the last specified place.<sup>b</sup>assumed and unadjusted<sup>c</sup>The errors in  $T_2$  are computed from the assumed error in  $T_1$ , 150 K, and that found for  $\Delta T$ .

normalized to the means of observations within  $0^{\circ}.02$  of the quadratures; the light curves have been arbitrarily shifted against the  $y$  axis to fit on the figure, but the scale has been preserved. The derived value for  $T_2$  (the temperature of the less massive star, and that eclipsed at secondary minimum) is larger than  $T_1$  so that the secondary minima in the calculated LC light curves in all passbands are slightly deeper. V728 Herculis is therefore a W-type system.

The absolute parameters of the system are summarized in Table 8.

## 5. DISCUSSION

The values for  $i$ ,  $T_2$ ,  $\Omega_1$ , and mean stellar radii  $R_1$  and  $R_2$  in the present solution are similar to those obtained by Samec (1990), but  $q$  is considerably different, as one could expect for a partial eclipse system light curve when radial velocities are not available. Consequently, since the contact parameter,  $f$ , sometime referred to as the "fill factor," is strongly dependent on the mass ratio, it also differs considerably from Samec's (1990) result. As used here,

$$f = \frac{\Omega_i - \Omega_o}{\Omega_i - \Omega_o}, \quad (1)$$

where the subscripts  $i$  and  $o$  refer to the inner and outer Lagrangian surface potentials. It should be noted that in this usage,  $0 \leq f \leq 1$ , unlike the quantity usually referred to as the "fillout factor," which has the value 1 at the inner Lagrangian surface, and 2 at the outer. In the present case it is found that:  $\Omega_i = 2.1787 \pm 0.0132$ ,  $\Omega_o = 2.0642 \pm 0.0105$ , and with  $\Omega = 2.0969 \pm 0.0100$  (all errors here are mse's), so that  $f = 0.714 \pm 0.112$ . This value is very large and suggests a substantial degree of contact between the components, despite the large apparent temperature difference. It implies a relatively advanced state of merger. The mass ratio  $q = 0.179 \pm 0.002$  and the masses are found to be  $M_1 = 1.654 \pm 0.037$ ,  $M_2 = 0.295 \pm 0.009$ . Recent work by Rasio (1995) and by Rasio & Shapiro (1994, 1995) suggests that contact systems with deep convective envelopes and mass ratios less than

### V728 Herculis Radial Velocities

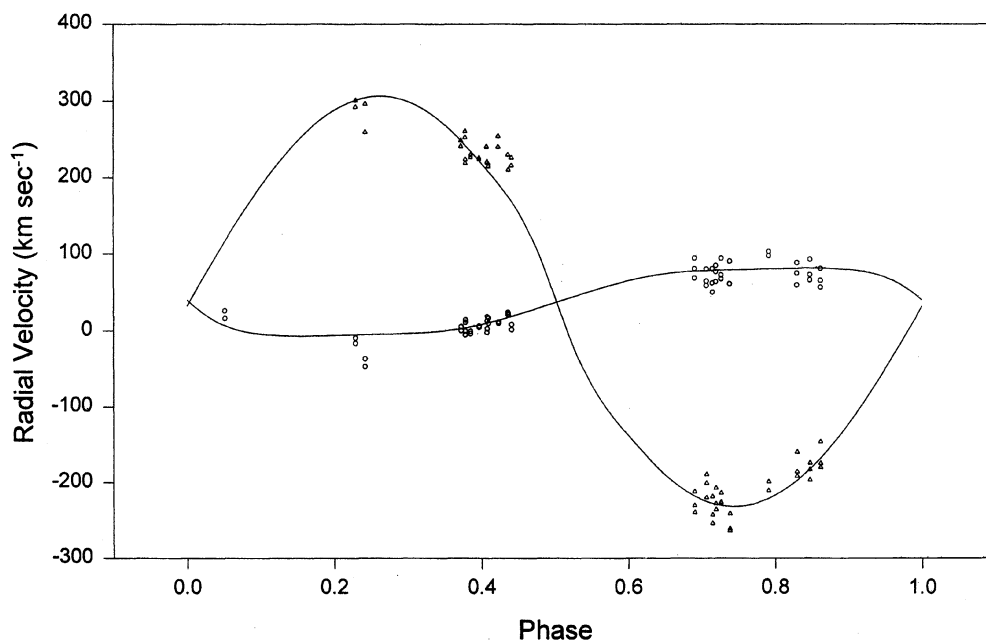


FIG. 1. Observed and computed radial velocity curves of the components of the field contact system V728 Herculis. The computed curves are for the final convective model, and were generated with the LC subprogram of WD93K93.

0.45 may be unstable and are entering into a final stage of merger. If that were the case for V728 Her, however, the O–C curve would show definite evidence of changing period. As can be seen in Fig. 3, however, this does not appear

to be the case. Therefore, either the system does not have a deep convective envelope, and we are deceived by the apparently better fit for the convective case, or there is some, as of yet, unknown condition or circumstance that is stabilizing

### V728 Herculis Light Curves

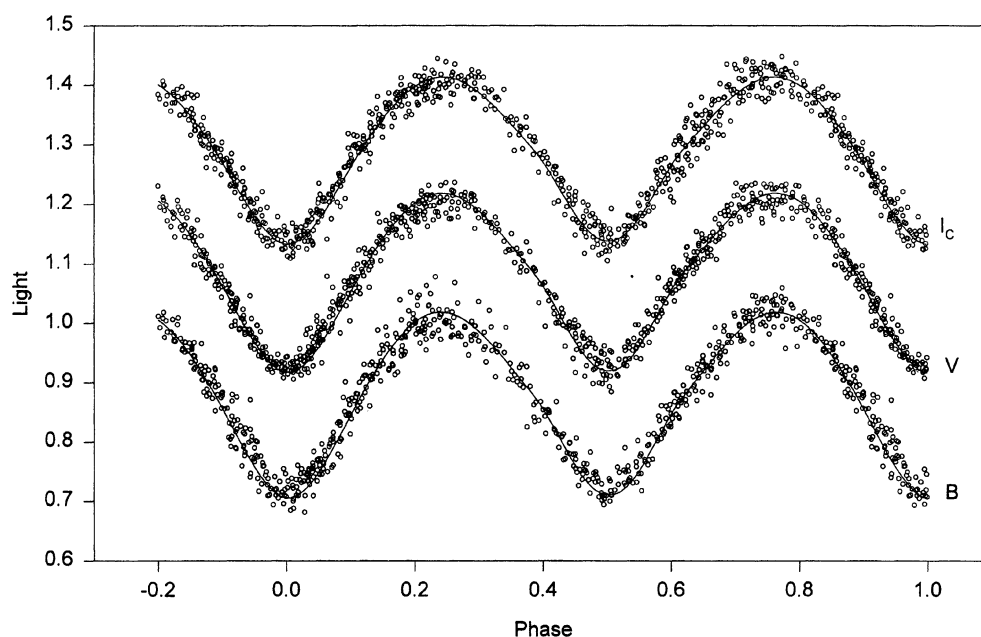


FIG. 2. Observed and computed light curves of the V728 Herculis system. The computed curves are for the final, convective model, and were generated with the LC subprogram of WD93K93.

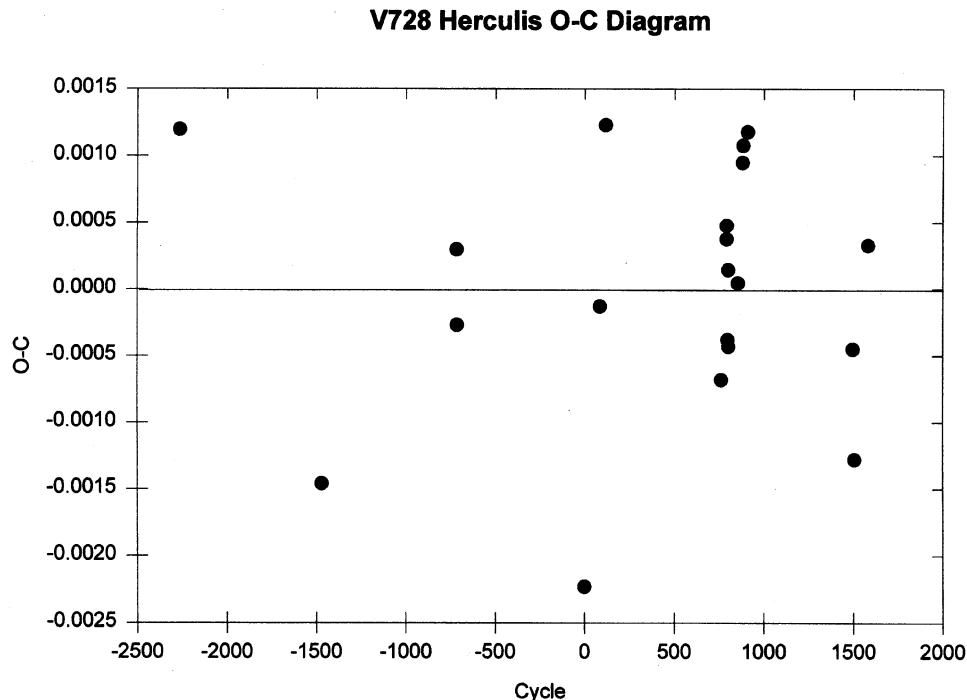


FIG. 3. The O-C curve of V728 Herculis computed with the ephemeris of Eq. (3). The flatness of the curve does not appear to support the idea that the system is currently unstable to collapse and is entering the rapid merger stage. See text for details.

the system. It is interesting to note that if the system were centrally condensed, but with properties given in Tables 6 and 8, the value of the primary star's dimensionless radius of gyration as computed from Rasio (1995) Eq. (1),  $k_1^2=0.16$ , which would place it on the stable side of his Fig. 1.

V728 Herculis lies in the region of the  $(B-V)$  vs  $\log P$  plot of Rucinski (1994, Fig. 1) near the systems H235 in NGC 752 (Milone *et al.* 1995a) and EV Cnc in M67 (Gilliland *et al.* 1991; where it is identified as Eggen-Sandage Star III-2). The light curve of EV Cnc, although shallow, has much stronger asymmetries, however. It would be interesting to see an analysis of the latter system, but as with most partial eclipse light curves, radial velocity data are crucial to modeling success.

Bradstreet's Binary Maker 2 program, was used to generate Fig. 4, the equipotential curves of the V728 Her system. Figure 5, generated by DT's ECPLLOT program, shows the system as it would appear to an observer near the system in the direction of the Earth at phase 0.25.

## 6. CONCLUSIONS

The small mass ratio, relatively low inclination, and early spectral type cause V728 Her to resemble the contact system, H235, in the open cluster NGC 752 (Milone *et al.* 1995a), which is apparently less than 2 Gyr old, if, as is argued, it is coeval with the cluster. Milone *et al.* (1995a) also argue that the dynamical evolution of H235 may have been accelerated to some degree by collisions in the cluster. In the case of V728 Her, that mechanism is not available, and its masses

TABLE 8. V728 Herculis absolute parameters.

Parameter (Unit)	Star 1	System	Star 2
$a$ ( $R_\odot$ )		3.180(24) <sup>a</sup>	
$i$ (degrees)		69.20(56)	
$f$		0.71(11)	
$M$ ( $M_\odot$ )	1.654(37)	1.949(43)	0.295(9)
$\sigma$ ( $\sigma_\odot$ )	5.48(54)		1.42(30)
$\mathcal{R}$ ( $\mathcal{R}_\odot$ )	1.784(15)		0.865(54)
$M_{\text{bol}}$	2.84(11)		4.31(23)
B.C.	0.02		0.02
$M_V$	2.86(11)	2.60(11)	4.31(22)
$(V - M_V)^b$		8.24(11)	
$r_{\text{uncorr.}}$ (pcs)		445(25)	
$(B - V)$		0.41(1)	
$E_{\text{B-V}}^c$		0.03:	
$(B - V)_0^c$		0.38:	
$(V - M_V)_0^d$		8.15:	
$r_{\text{corr.}}$ (pcs)		427:	

<sup>a</sup> Uncertainties in this table are mean standard errors.

<sup>b</sup>  $\langle V \rangle = 10.84$ ,  $\langle B - V \rangle = 0.41$  from Samec (1990).

<sup>c</sup>  $(B - V)_0$  from Popper (1980), whence  $E_{\text{B-V}} = 0.03$ .

<sup>d</sup> Assuming  $A_V = 3.1E_{\text{B-V}}$ ,  $A_V = 0.09$  so that  $V_0 = 10.75$ .

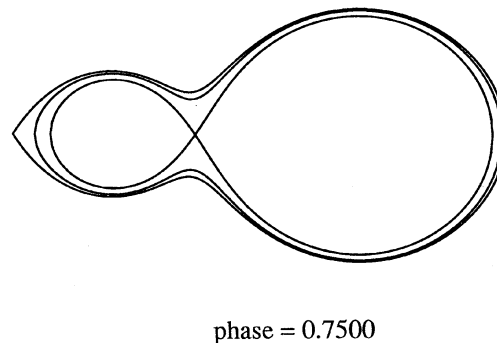


FIG. 4. The equipotential curves of the V728 Her system, created with Bradstreet's Binary Maker 2 program.

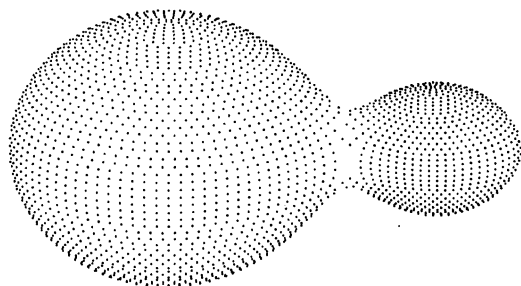


FIG. 5. The binary star system V728 Herculis as it would appear to an observer near the system at phase 0.25, generated by DT's ECPLLOT program.

and contact parameter are larger, so the evolutionary state may not be comparable. On the other hand, its large contact parameter makes this system a candidate for instability according to the criteria discussed by Rasio (1995); yet the O–C curve does not support such a conclusion.

Radial velocity and photometric data obtained over several seasons have been analyzed and solutions found for this field contact binary. The periods and epochs of Agerer *et al.* (1988) and Samec & Butcher (1989) are essentially confirmed. The latest University of Calgary version of the Wilson–Devinney program, WD93K93, was used for the analyses, and the upgraded simplex version of this code, LC93KS, was used to test model uniqueness. The modeling results indicate that V728 Herculis is a W-type W UMa sys-

tem with a significant temperature difference between components. The degree of over-contact as indicated by the contact parameter  $f$  is large for such a system, which is believed by many investigators to be in an earlier state of development than an A-type system. The mass ratio is based on combined radial velocity and photometric modeling and differs significantly from the photometric mass ratio determined by Samec (1990). The conclusions indicate that the system deserves further investigation, especially continued photometry in a large range of passbands, and additional modeling. Finally, there is the question of chemical composition, which may have an effect on modeling. In the present case, we have performed all modeling under the assumption of solar metallicity. Since the WD93K93 curve attempts to fit the radiometric properties of binaries as well as their geometric properties, the correct composition may impact the optimum solution.

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