Rail car fleet design: Optimization of structure and size

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Abstract

We develop a model to determine the optimal structure and size of a rail car fleet at a chemical company under uncertainty in demand and travel times as well as substitution between rail car types. First, we formulate an MILP model that accounts for the substitution relations between the types and minimizes the total direct rail car cost under given rail car availability constraints and a predefined maximum number of types. Second, based on the fleet structure obtained by the MILP model, the fleet size is computed by using an approximation from inventory theory that considers the existing uncertainties. Compared to the current approach of the rail car fleet management team, the model produces a reduction in safety stock of 120 rail cars and thus direct cost savings of 8% as well as further indirect cost savings due to a smaller number of rail car types, which reduces the switching effort of the rail cars on the storage tracks.

Keywords:

fleet management, fleet structure, inventory, safety stock

1. Introduction

In the chemical industry, rail cars represent an important means of transportation. Due to safety regulations many products are not allowed to be transported on the road. Moreover, rail cars can carry larger volumes than trucks. The product poses minimum requirements on a rail car with respect to material, valve model, heating, etc. The combination of these characteristics specifies a certain rail car type and determines its cost. Types with higher quality characteristics can be used as substitutes for lower ones and thus are more flexible.

At the company, which motivated this research, the task of the rail car fleet management team is to secure the supply with rail cars of an appropriate type while at the same time solve the trade-off between (i) minimizing the direct cost for rail cars and (ii) minimizing the number of different rail car types. The latter aspect is relevant because the smaller the set of rail car types, the easier it is to access a requested type on the storage tracks due to

Preprint submitted to Elsevier

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a sorted parking strategy. As the number grows, space limitations require a chaotic parking strategy, which increases the switching effort and thus causes higher indirect costs. Further, the smaller the set of types, the lower the required safety stock due to a larger risk pooling effect. These benefits have to be traded off against the higher costs for more flexible types.

Over the last decade, the fleet management team has invested considerable effort to reduce the overall cost and free up storage space on the site. In a first analysis of the rail car fleet, old and seldomly used types that could be easily replaced by others have been discarded. Thus, the number of so-called standard rail cars (which we will be focusing on in this paper) has been reduced from approx. 2600 to 1800. Similarly, the number of standard rail car types has been reduced from approx. 100 to 60. These tremendous improvements have been made possible through hard work, but without any support from sophisticated mathematical models. To realize further improvement and be well prepared for the future, management felt that such models are required.

In the light of the growing future business trend the rail car demand is expected to increase as well. Despite the already implemented reforms, it will not be feasible to meet these demands with the current order-filling strategy and structure of the rail car fleet. The limited space on the site simply hampers an increase in the number of cars. Moreover, as part of a new strategy the company considers to increase its rail car ownership. This is based on the insight that owning a rail car is much cheaper than leasing one, if the usage period is sufficiently large. It takes about 10 years for the investment in a rail car to amortize. In order to make a suggestion with regard to which types of rail cars to buy (*fleet structure*) and in which quantity (*fleet size*), a second thorough quantitative analysis is required.

In this paper we present the outcome of this second analysis. Together with the fleet management team mathematical models have been developed that take into account the existing trade-offs and are used as decision support for designing the rail car fleet. This analysis is based on a new approach which combines mixed integer linear programming (MILP) models supporting substitution between the different rail car types with techniques used in inventory management theory to derive safety stocks in order to account for the existing uncertainties in demand and travel times of the rail cars.

The remainder of the paper is structured as follows. In Section 2, we briefly review the literature. The mathematical models are developed in Section 3. In Section 4, we apply the models to the real-world problem data. We conclude the paper in Section 5.

2. Literature review

Research on fleet sizing and structuring started in the 1950s with deterministic models. Dantzig and Fulkerson (1954) determine the minimal number of tankers to meet a fixed schedule. Further research by Gertsbach and Gurevich (1977) and Ceder and Stern (1981) has led to the derivation of the well-known fleet size formula. While these deterministic models emphasize the spatial structure of the problem, the stochastic nature of demand and travel times is neglected.

The latter aspects are considered in Koenigsberg and Lam (1976), Parikh (1977), and Papier and Thonemann (2008), who make use of queueing models to account for the uncertainties. Koenigsberg and Lam (1976) analyze the effect of the fleet size on the mean delay time in a gas vessel cycle between two sea ports. Based on an M/G/c queueing model, Parikh (1977) determines the optimal structure of a rail car fleet such that the service levels of all rail car types are nearly identical. Papier and Thonemann (2008) use an $M^x/G/c/c$ queueing model to explicitly account for customer order batching as well as seasonal demand.

List et al. (2003) formulate a large stochastic programming problem to determine the optimal fleet size under uncertainty in demand, travel times, and further operational aspects. Due to its complexity, the model easily becomes intractable for large problems, however.

Besides the pure fleet-sizing problem, several authors address the interdependency between the fleet size and the management of empty and loaded vehicle flows (see, e.g., Beaujon and Turnquist (1991), Cheung and Powell (1996), Wu et al. (2005), and references therein). Due to the complexity of the problem, most works assume either deterministic demand or deterministic travel times in order to obtain a solution. For the solution of a stochastic version of the problem, Köchel et al. (2003) propose a simulation optimization approach.

In the application that motivated this research, we also face stochastic demand and travel times as well as customer order batching. In addition and in contrast to the previous works (except for Wu et al. (2005)), substitutions between different rail car types are possible. This prevents us from directly applying any of the existing above-mentioned approaches.

Through the substitution aspect the problem is also related to transshipment models in inventory theory. The possible upward substitution between different rail car types can be interpreted as a unidirectional lateral transshipment as explained and analyzed in, e.g., Axsäter (2003) and Olsson (2010). For a general overview on transshipment models see Paterson et al. (2011). However, those models do not readily fit either for the following reasons. They make certain assumptions with respect to the lead (travel) times or the demand arrival process, which are not satisfied in our real-world problem setting. Most importantly, however, they rely on enumerative solution methods, which for our real-world problem size with up to 20 substitution possibilities, are prohibitively time consuming.

Therefore, we develop a different solution approach, which is easier to solve in our view, but still accounts for all relevant problem aspects. We use a combination of deterministic MILP models and stochastic models originating in inventory management theory. In the deterministic part of our model we account for the substitution possibilities. The outcome of this first solution step is the fleet structure. Based on the deterministic solution, the existing uncertainties concerning demand and travel times are dealt with in a second step, the fleet-sizing part of the model, which is based on an approximation from inventory theory.

3. Model

3.1. Problem description and notation

The goal of the analysis is to provide a suggestion for the "optimal" design of the rail car fleet. As such, the planning problem is tactical/midterm in nature rather than operational/shortterm. The focus is not on how to reach a close-to-optimal solution as fast as possible and provide a detailed implementation plan. It is on what the "optimal" solution looks like in the first place. Therefore, the current rail car fleet design of the company can be neglected in the analysis, i.e. we basically assume that we can design the fleet from scratch.

When deciding about the structure and size of the fleet, the fleet management team has to consider and trade off various aspects. First, the supply of the appropriate rail cars needs to be secured. In terms of the mathematical model, this aspect translates into the requirement that (i) all orders within the planning horizon are to be satisfied in a deterministic model formulation or (ii) a high level of service needs to be provided in a stochastic inventory model formulation.

Second, this service is to be achieved at the lowest possible cost. For each rail car of a certain type, we have a specific direct cost. This cost is incurred once for the entire planning horizon, if the rail car is used at all irrespective of the actual timespan that it is in use. This kind of modeling appropriately reflects the majority of the existing leasing contracts.

Third, not only the direct costs for rail cars are to be minimized, but also the number of different rail car *types* is to be kept at a low level in order to save indirect costs. Due to space restrictions, a large number of different types requires a chaotic parking strategy on the storage tracks. This causes a considerable switching effort for providing rail cars of a certain type. A reduction in the set of types to only a few would enable a sorted parking strategy where each type is parked on a separate track facilitating the handling. In addition, a type reduction also has a positive effect on the direct rail car cost. The risk pooling effect can be exploited to a larger extent, which results in a lower overall safety stock requirement.

Fourth, substitution between different rail car types is feasible. The transported product poses minimum requirements on certain rail car characteristics (material, valve model, heating, etc.). These characteristics define a rail car type. Types with higher quality characteristics can be used as substitutes for lower ones and thus are more flexible. On the other hand, a more flexible type is more expensive, in general. This flexibility aspect is very important when it comes to the structuring and sizing of the fleet.

Fifth, due to market restrictions not all rail car types are available in an unlimited quantity. Some types are no longer produced or are very expensive to produce. Therefore, only the number of rail cars currently circulating in the market is considered "available".

Before we develop an optimization model that takes all of the above-mentioned aspects into account, we first describe the current planning and execution approach of the fleet management team. This forms the benchmark for the evaluation of the more sophisticated mathematical model, which we develop in a second step. We use the following notation:

Indices and sets

$a \in \mathcal{A}$	orders
$d\in \mathcal{D}$	days in the planning horizon
$k \in \mathcal{K}$	rail car types
$\mathcal{A}_d \subset \mathcal{A}$	subset of orders that are active on day d (new orders plus orders in transit)
$\mathcal{A}_k \subset \mathcal{A}$	subset of orders that require rail car type k
$\mathcal{K}_a \subset \mathcal{K}$	subset of rail car types that can fill order a

Parameters	
T	number of rail car types
N_{dk}	number of rail cars of type k used on day d (derived from problem data)
\overline{x}_k	maximum number of available rail cars of type k
c_k	cost per unit and year of rail car type k
α_k^{target}	non-stockout probability service-level target of type k
μ_{SF_k}	mean of the shortfall random variable of type k
σ_{SF_k}	standard deviation of the shortfall random variable of type \boldsymbol{k}
Variables	
$B_k \in \mathbb{R}$	base-stock level of rail car type k
$S_k \in \mathbb{R}$	safety stock level of rail car type k
$n_{dk} \in \mathbb{N}_0$	number of rail cars of type k used on day d
$x_k \in \mathbb{N}_0$	number of rail cars of type k that are used in the planning horizon
$y_{ak} \in \{0, 1\}$	rail car type k is assigned to order a
$z_k \in \{0, 1\}$	rail car type k is used/activated

Random variable

 SF_k shortfall random variable of rail car type k

Note that we define an order a as a request for one rail car, which is characterized by a minimum requirement for a certain rail car type k plus a start and end day. During these points in time the order is considered "active" and a rail car of a compatible type needs to be assigned to it. That means, we manipulate the (historical or forecasted) order data, if required; we redefine original orders that request more than one rail car per order to fit our definition. Given this definition the above-specified sets are easily obtained. Of note is that the substitution relations between the different rail car types enter the model through set \mathcal{K}_a .

3.2. Optimization without substitution – Current approach

In the current situation, the fleet management team mainly pursues a strategy of assigning to each order exactly the requested rail car type. Only in very few instances, a substitute is assigned, if the requested type is not available. The reason is that, prior to the development of the model described in this paper, the existing substitution relations have not been formalized. Therefore, this information has not been accessible to the dispatcher. The dispatcher decided whether to assign a substitute type and also which one based on experience. Given the large number of possible substitutions (as it has turned out during the modeling phase), this task could not be optimally performed manually. Hence, substitutions have not been used to a large extent. As such, we can consider the current planning and execution approach in a mathematical/formal way as an optimization model without any substitution. That means, in our deterministic model we determine the optimal number of rail cars for each type k simply as

$$x_k^{NoSubst} = \max_{d \in \mathcal{D}} \{N_{dk}\}$$
(1)

where N_{dk} denotes the number of rail cars of type k that are in use on day d. For each day of the planning horizon \mathcal{D} , this quantity can be easily obtained directly from the (historical or forecasted) order data. The total direct costs for all types are computed as

$$C^{NoSubst} = \sum_{k \in \mathcal{K}} c_k \cdot x_k^{NoSubst}.$$
(2)

Note that in this approach no upper bounds on the rail car availability of a certain type can be taken into account. Since all orders have been filled with more or less no substitution in the past, however, we assume that we can neglect this aspect in this simple computation. Things are different when we want to exploit substitution benefits. Then, the availability constraints become highly relevant, because orders for various types may be redirected to only one specific type, whose maximum availability needs to be considered. This is done in the models in Section 3.3.

3.3. Optimization with substitution

As explained in Section 3.1, the overall objective is to minimize both the direct rail car cost as well as the number of rail car types causing switching/complexity costs on the storage tracks. Hence, we actually face a multi-criteria optimization problem. If it was possible to exactly quantify the complexity reduction stemming from a type reduction in terms of cost, we could simply minimize the total cost (direct rail car cost plus complexity cost) as a single objective and find the optimal solution. However, in our real-world problem it is difficult to derive a resilient relationship between the number of rail car types and the resulting complexity cost. Therefore, we do not directly include the latter aspect as a cost term in the objective function, but pursue a different approach. In the objective function we only minimize the direct rail car costs. We consider the number of different types as an additional constraint in our optimization model. By varying this number within certain bounds, we can present the management with a cost curve as a function of the number of rail car types. They can then choose a favorable combination on this curve.

In order to generate this cost curve, we need to specify upper and lower bounds on the number of rail car types T. A natural upper bound is easily found as the sheer number of different types and is denoted as T_{max} . In order to find a lower bound, T_{min} , that complies with the availability constraints on the different types and ensures that all orders in the planning horizon are filled, we formulate an optimization problem in the following section. Afterwards, we present the cost minimization model for a given number of types.

In neither of these two models in Section 3.3.1 do we consider the uncertainty in the demand and travel times. These aspects are addressed afterwards in Section 3.3.2. That means, our approach is a simplified and approximate sequential one. First, in the fleet structuring phase (Section 3.3.1), we employ a deterministic model. Second, when it comes to the fleet sizing for a *given* structure, we take the uncertainties into account (Section 3.3.2).

3.3.1. Fleet structure

Rail car type minimization. We formulate a deterministic optimization model for minimizing the number of required rail car types. The model takes the specified substitution relations between the different types and certain upper bounds on their availability into account, but no costs. If we neglect the availability constraints, the problem is a simple "set covering problem". Since the availability constraints are highly relevant, however, we obtain the minimum number of types as the solution to:

 $z_k \ge y_{ak}$

$$\mathbf{P}^{type} \quad \min \quad T_{min} = \sum_{k \in \mathcal{K}} z_k \tag{3}$$

s.t.
$$\sum_{k \in \mathcal{K}_a} y_{ak} = 1$$
 $\forall a \in \mathcal{A}$ (4)

$$\forall a \in \mathcal{A}, \forall k \in \mathcal{K}_a \tag{5}$$

$$n_{dk} = \sum_{a \in \mathcal{A}_k \cap \mathcal{A}_d} y_{ak} \qquad \qquad \forall d \in \mathcal{D}, \forall k \in \mathcal{K}$$
(6)

$$n_{dk} \le x_k \qquad \qquad \forall d \in \mathcal{D}, \forall k \in \mathcal{K} \tag{7}$$
$$x_k \le \overline{x}_k \qquad \qquad \forall k \in \mathcal{K} \tag{8}$$

$$\begin{aligned}
& w_k \leq w_k \\
& y_{ak} \in \{0, 1\} \\
& z_k \in \{0, 1\}
\end{aligned}$$

$$\begin{aligned}
& (0) \\
& \forall a \in \mathcal{A}, \forall k \in \mathcal{K}_a \\
& \forall k \in \mathcal{K}.
\end{aligned}$$

$$(10)$$

In the objective function, the number of different rail car types is minimized. Constraint (4) ensures that exactly one rail car of a specific type k is assigned to each order a for its entire active duration. After the active period of an order, one and the same rail car of type k may well be assigned to another order that starts at a later point in time. Due to this constraint all orders are satisfied by a rail car. This is done directly from the start day of the order. Thus, backordering is not allowed in this deterministic model. If type k is assigned to order a, z_k is set to 1 by constraint (5) indicating that type k belongs to the set of required types in this solution. Constraint (6) computes the number of required rail cars of type k on day d and assigns it to n_{dk} . This constraint considers only the active orders on each day $a \in \mathcal{A}_d$ and only those orders that can be satisfied by rail car type k, which are summarized in set \mathcal{A}_k . The total number of required rail cars of type k over the entire planning horizon, x_k , needs to be at least as large as n_{dk} for all days within the planning horizon. This is reflected by constraint (7). Constraint (8) ensures that the total number of required rail cars of type k complies with the maximum availability of that type, \overline{x}_k . The remaining two constraints specify the binary character of the decision variables.

Given the solution to \mathbf{P}^{type} in the form of T_{min} , we know that a feasible solution exists for a cost-minimization model with the same constraints plus one additional constraint in the form of an upper bound on the number of possible rail car types T as long as $T \geq T_{min}$. This model is formulated next. Total direct cost minimization. For a given upper bound on the number of rail car types $T_{min} \leq T \leq T_{max}$ we find the cost optimum as the solution to:

 $z_k \geq y_{ak}$

$$\mathbf{P}^{cost} \quad \min \quad C = \sum_{k \in \mathcal{K}} c_k \cdot x_k \tag{11}$$

s.t.
$$\sum_{k \in \mathcal{K}_a} y_{ak} = 1$$
 $\forall a \in \mathcal{A}$ (12)

$$\forall a \in \mathcal{A}, \forall k \in \mathcal{K}_a \tag{13}$$

$$\sum_{k \in \mathcal{K}} z_k \le T \tag{14}$$

$$n_{dk} = \sum_{a \in \mathcal{A}_k \cap \mathcal{A}_d} y_{ak} \qquad \forall d \in \mathcal{D}, \forall k \in \mathcal{K}$$
(15)

$$n_{dk} \le x_k \qquad \qquad \forall d \in \mathcal{D}, \forall k \in \mathcal{K} \tag{16}$$

$$x_k \le \overline{x}_k \qquad \qquad \forall k \in \mathcal{K} \tag{17}$$

$$y_{ak} \in \{0, 1\} \qquad \forall a \in \mathcal{A}, \forall k \in \mathcal{K}_a \qquad (18)$$
$$z_k \in \{0, 1\} \qquad \forall k \in \mathcal{K} \qquad (19)$$

The objective function of \mathbf{P}^{cost} minimizes the total direct cost of the number of rail cars of type k that are required to satisfy all orders in the planning horizon. In comparison to \mathbf{P}^{type} , we have one additional constraint (14), which ensures that the number of chosen rail car types does not exceed the predetermined upper bound T.

Given the solution to \mathbf{P}^{cost} for all $T_{min} \leq T \leq T_{max}$, we can generate the required cost curve, which is used as decision support. An example of such a cost curve is depicted in Figure 1 of the "Application" Section 4.

3.3.2. Fleet size – Safety stock approximation

The above-described models are purely deterministic. They do not account for any uncertainties in the number of rail cars demanded on a certain day or the travel times, i.e. the timespan, for which the rail cars are in use. One way to accommodate these aspects is to solve \mathbf{P}^{type} and \mathbf{P}^{cost} for multiple scenarios with different demand and travel-time patterns and assign probabilities to these scenarios. Thus, a certain buffer could be determined for each rail car type. This approach would be very time consuming, however.

Alternatively, the problem can be analyzed from an inventory optimization perspective and a safety stock can be computed for each rail car type that is part of the "optimal" fleet structure obtained from the deterministic models of Section 3.3.1. This is the approach we choose. In the inventory theory terminology, the described situation that each rail car type faces in isolation of the others can be interpreted as an inventory system with stochastic demand and lead times that is controlled by a base-stock policy where backordering is allowed. The orders for rail cars are the demands. (Note that in this case we talk about orders in the original order data sense where each order may represent a request for more than one rail car.) Each time an order is filled by rail cars of a specific type, these rail cars are in use for a certain period of time. This timespan represents the lead time of the replenishment order in the inventory system and the size of the replenishment order corresponds to the number of requested rail cars. Since each rail car order triggers a replenishment order of the corresponding size, the inventory control policy is a base-stock policy. Depending on the different destinations specified in the orders, the travel times of the rail cars of one and the same type might vary significantly. That means, the lead times in the inventory system are stochastic and orders may even cross. To illustrate this, consider the following situation: On day 10, 5 rail cars of type k are requested for a trip to a destination that is 600 km away. On day 11, 7 rail cars of the same type k are requested for a trip to a destination that is only 50 km away. Given that all other circumstances are identical, the 7 rail cars that were "replenished" at a later point in time (day 11) will arrive earlier than the once replenished on day 10.

For the computation of the base-stock levels for the rail car types we rely on approximations in two respects. (i) In order to reduce the computational complexity, we do not explicitly incorporate the substitution relations between the different types into the inventory model. (Otherwise, we would have to solve a transshipment problem with more than 20 locations in our example in Section 4. Neither an appropriate transshipment model formulation for our specific problem setting nor an efficient solution algorithm is currently available in the literature, see Section 2.) Instead we consider T separate optimization problems, but take the substitution possibilities into account implicitly through an appropriate data aggregation. When we estimate the parameters that enter the inventory model, we consider the assignment of rail car types to orders, which has been obtained as the optimal solution to \mathbf{P}^{cost} . Consequently, orders for a specific rail car type k that are filled by type l in the optimal solution, will be regarded as order requests for type l when it comes to the parameter estimation for the inventory model. That means, we take the fleet structure as a given.

(ii) For the base-stock level optimization of each rail car type k, we follow an approach similar to Bradley and Robinson (2005) that accounts for order crossover. The relevant input to the inventory model is the number of rail cars that are in use at a certain point in time t. Let us denote this number as the shortfall, SF_k^t . The stationary distribution of the shortfall random variable is denoted as SF_k . In their approach, Bradley and Robinson (2005) first perform two estimations, one for the distribution parameters (mean and standard deviation) of the number of outstanding orders and one for the distribution parameters of the quantity of an order. Then, they combine these estimates to obtain the distribution parameters for SF_k and assume that it follows a normal distribution. In order to avoid this kind of double estimation problem, we estimate the mean and standard deviation of SF_k , μ_{SF_k} and σ_{SF_k} , directly from the problem data and assume that it follows a normal distribution.

For a given service-level target α_k^{target} of the type "non-stockout probability", we find the optimal base-stock level B_k^* for rail car type k as

$$B_k^* = \min\left\{B_k \mid \Pr\{SF_k \le B_k\} \ge \alpha_k^{target}\right\}.$$
(20)

The safety stock is derived from B_k^* as

$$S_k = B_k^* - \mu_{SF_k}.\tag{21}$$

In absence of any substitution, we simply consider T_{max} separate inventory optimization problems *without* aggregating or regrouping any of the problem data. The base-stock levels and the required safety stocks are obtained accordingly.

4. Application

4.1. Data and implementation

We apply the model of Section 3.3 to a real-world problem. Our objective is to quantify the improvement potential compared to the current approach of the fleet management team (Section 3.2), because this is what the company is most interested in. From a theoretical perspective, it would also be interesting to analyze the performance of our two-step approximate approach relative to others. However, since no other appropriate modeling and solution approaches for this particular problem setting exist in the literature (as explained in Section 2), we postpone such an analysis to future research.

Due to confidentiality reasons, we cannot disclose the problem data in detail. We consider order and return data on a daily basis for one year, which is our planning horizon. We conduct the analysis on the most recent historical data, i.e. data from 2011. Based on the rail car characteristics the fleet management team has specified 62 different rail car types named "ST00" to "ST61". In 2011, only 41 of these 62 types have actually seen order requests. Therefore, when we discuss the situation with no substitution in the following sections, we refer to these 41 types. Two types have (estimated) availability constraints of 70 and 400 units, i.e. types "ST19" and "ST49". Moreover, for each rail car type a list with possible substitute types has been prepared by the fleet management team. For some types up to 24 substitutes are available. The direct cost per rail car type varies between 3,650 and 14,000 per year. For the stochastic inventory model we assume a non-stockout

probability target of 95%.

We have implemented all models in GAMS 23.7 (cf. Brooke et al. (1992) or Bussieck and Meeraus (2004)) on an Intel x64 with 3.3 GHz and 8 GB memory. \mathbf{P}^{type} and \mathbf{P}^{cost} have been solved with GAMS/CPLEX. \mathbf{P}^{type} has 229,366 constraints and 133,849 variables, 20,512 of which are binary. It has taken 17 seconds to obtain the solution T_{min} . \mathbf{P}^{cost} has 249,880 rows constraints and 133,912 variables, 20,574 of which are binary. A solution has been obtained for each T after 136 seconds. The small computing times could only be realized by exploiting some of the polylithic modeling and solution techniques described in Kallrath (2011). Initial solutions have been provided derived from a graph coloring problem, and the *mipstart* feature of GAMS/CPLEX has been used extensively. The GAMS/CPLEX presolve-techniques have further reduced the problem size to 27,551 rows and 20,519 binary variables. For the safety factor computation in the inventory model the inverse of the standard normal cumulative distribution function has been approximated according to Abramowitz and Stegun (1970).

4.2. Results

We start with the first step of our solution approach. We determine the fleet structure based on the deterministic models, \mathbf{P}^{type} and \mathbf{P}^{cost} , that take into account substitution and the exact order data of 2011. By solving \mathbf{P}^{type} with the data including the availability



Figure 1: Direct costs for different numbers of rail car types (based on \mathbf{P}^{cost})

constraints, we obtain a lower bound on the number of required rail car types of $T_{min} = 8$, which we call the "minimal pool". Without the availability constraints, the minimum set of rail car types consists of 6. This follows from solving the respective set covering problem. That means, the availability constraints require us to use (at least) two more types.

Even though the upper bound on the number of types is $T_{max} = 41$, we do not enumerate all feasible values between T_{min} and T_{max} to draw the cost curve, but proceed in a slightly different way. We start at T_{min} and compute the optimal solution to \mathbf{P}^{cost} with a stepsize of 1. Once the cost change is basically negligible for the last 5 *T*-values, we stop the computation. In this case, increasing the bound *T* no longer seems to result in any significant cost reduction. Figure 1 shows the resulting cost curve for the relevant region $8 \leq T \leq 23$.

For this particular setting, we observe a sharp cost drop when adding one additional rail car type to the minimal pool ($T_{min} = 8$). Table 2 illustrates the detailed composition of the rail car fleet resulting from the deterministic model. As T is increased from 8 to 9, type "ST29", which has a very low annual cost, is added to the fleet with a quantity of nearly 100. By almost the same extent, the much more expensive types "ST41" and "ST49" are reduced. This causes the major cost reduction. A further extension of the rail car type set only leads to minor additional cost savings. For T = 10, type "ST01" is added and partly substitutes type "ST22". The cost difference between these two types is not as large as the previous one, however. Therefore, the cost drop is not as pronounced.

When it comes to deciding about the ideal number of rail car types, we need to keep in mind that the cost curve only represents the direct rail car costs. Complexity costs due to an increased switching effort on the storage tracks as the number of rail car types grows are not included. These costs need to be considered as well. In this particular real-world problem setting, no resilient cost estimates are available for the complexity cost as explained in Section 3.3. Therefore, we need to rely on a more subjective judgement by management in this respect. Based on the direct cost savings illustrated in the cost curve and their experience, management came to the conclusion that the savings resulting from an increase to T = 10 are easily outweighed by the increased switching effort of having more rail car types on the storage tracks. Therefore, the best set of rail car types is found to consist of 9 types. (Note that in situations where a complexity cost factor per additional rail car type can be derived, this cost component can be accounted for directly in the objective function of the MILP in order to avoid such a subjective decision.)

Rail car type Rental cost		Number of rail cars (for different T)				
(Availability)	per car and year []	8 (Minimal pool)	9	10		
ST00	11,735.20	12	12	12		
ST01	4,562.50	0	0	32		
ST02	6,267.05	9	9	9		
ST13	5,066.58	35	35	35		
ST19(70)	6,123.88	27	12	12		
ST22	6,009.34	271	286	254		
ST29	4,562.50	0	96	96		
ST41	9,904.21	977	945	945		
ST49 (400)	$11,\!424.39$	202	138	138		
ST51	9,719.95	16	16	16		
		1549	1549	1549		

Table 2: Optimal fleet structure and size for varying number of rail car types (based on \mathbf{P}^{cost})

Based on the deterministic model finding that T = 9 is the favorable value for the number of rail car types, we continue with the second step of the solution approach, the fleet sizing under uncertainties. With the help of the stochastic inventory model we compute for each of the 9 rail car types the required number of rail cars including the safety stock requirement. In addition, we compare the results to the ones for the minimal pool, i.e. $T_{min} = 8$, and the current situation where no substitution possibilities are exploited.

Since the inventory model computations rely on the assumption of a normally distributed shortfall random variable, we first test the fit and accuracy of this assumption for T = 9. Note that for some rail car types ("ST00, ST02, ST51") very few different shortfall data points are available, which makes it hard to fit a theoretical distribution (see Table 3). On the other hand, the available shortfall realizations for those types do not differ by much, which causes a low variability and thus a low safety stock requirement. Therefore, the accuracy of the fit for those types is less relevant for the upcoming stock computation. For the other types with more available different data points, we find that the normal distribution approximates the shortfall reasonably well, in general (keeping in mind that the data quality is not 100% either). Figure 2 illustrates the empirical data and the normal approximation.

Rail car type	ST00	ST02	ST13	ST19	ST22	ST29	ST41	ST49	ST51
Number of realizations	2	2	7	7	52	30	126	14	3

Table 3: Empirical data - Number of different shortfall realizations per rail car type

Given this fit, we now turn to the comparison of the stock quantities and the cost based on the inventory model. By looking at the quantities only, we find that, obviously, in all settings the average number of rail cars in use is identical, which is reflected by the constant total pipeline stock. Only the safety stock quantities differ. For the minimal pool, we obtain the numbers summarized in Table 4. In total, a safety stock of 192 cars is required. If we perform the same calculation for T = 9, the total safety stock requirement increases slightly by 1 car (cf. Table 5). However, this small increase in the quantity is easily compensated by the fact that a cheaper type ("ST29") is now part of the feasible set. This different composition of the rail car type pool affects the cost of *both* the pipeline stock and the safety stock. Consequently, the total cost drops from 14.72m to 14.12m, i.e. by 4%. Three percent of this reduction is due to the lower pipeline stock cost and 1% due to the safety stock cost.

In the situation without any substitution, the safety stock requirement amounts to 312 cars. Hence, by restricting the set of rail car types to 9 we can achieve a quantity reduction of approx. 120 cars. In terms of cost, it translates into a reduction of 1.32m or 8%.

Rail car type	Rental cost per car []	Pipeline stock	Safety stock	Total stock	Total cost []
ST00	11,735.20	12	3	15	176,028.02
ST02	6,267.05	8	1	9	56,403.45
ST13	5,066.58	33	4	37	187,463.62
ST19	6,123.88	23	5	28	171,468.69
ST22	6,009.34	249	30	279	$1,\!676,\!604.57$
ST41	9,904.21	910	110	1020	10,102,296.31
ST49	11,424.39	156	36	192	2,193,482.28
ST51	9,719.95	13	3	16	$155,\!519.20$
		1404	192	1596	14,719,266.14

Table 4: Optimized fleet structure and size for T = 8, i.e. minimal pool (based on inventory model)

Rail car type	Rental cost per car []	Pipeline stock	Safety stock	Total stock	Total cost []
ST00	11,735.20	12	3	15	176,028.02
ST02	6,267.05	8	1	9	$56,\!403.45$
ST13	5,066.58	33	4	37	$187,\!463.62$
ST19	6,123.88	10	3	13	$79,\!610.46$
ST22	6,009.34	262	31	293	1,760,735.27
ST29	4,562.50	81	20	101	460,812.50
ST41	9,904.21	851	117	968	$9,\!587,\!277.28$
ST49	$11,\!424.39$	134	11	145	$1,\!656,\!536.10$
ST51	9,719.95	13	3	16	$155,\!519.20$
		1404	193	1597	14,120,385.90

Table 5: Optimized fleet structure and size for T = 9 (based on inventory model)

Furthermore, by only having 9 different rail car types we reduce the complexity for the management of the rail cars on the storage tracks. Due to the very small number of different types, a sorted parking strategy can be implemented. This will lead to a decrease in the switching effort and thus further cost reductions in addition to the above-mentioned ones.

Finally, the analysis shows that the largest numbers of rail cars are required for types "ST22" and "ST41". Consequently, when it comes to a suggestion for purchasing rail cars, these are the two types to consider. Assuming that the general distribution of the orders for

the different types remains mostly unchanged in the future, at least the average quantities, i.e. the pipeline stock numbers, can be bought without having to worry much about a poor utilization of the rail cars.

5. Conclusions

In this paper we have developed a two-step model to determine the optimal structure and size of a rail car fleet at a chemical company under uncertainty in demand and travel times as well as substitution between rail car types. First, we have used an MILP formulation to determine the optimal fleet structure by minimizing the total direct rail car cost under substitution, given rail car availability constraints and a predefined maximum number of types. Second, uncertainties are accounted for in the subsequent fleet-sizing phase by employing an approximation from inventory theory. This approach is easily implementable and applicable to large problems, since it avoids an explicit and complicated consideration of the substitution possibilities directly in the stochastic inventory model. Even though the developed model surely contains certain company-specific aspects, the general approach of dividing the entire problem into two more easily manageable subproblems is widely applicable.

Compared to the current planning and execution approach of the rail car fleet management team, the presented model produces a reduction in safety stock of approx. 120 rail cars and thus direct cost savings of 8%. Moreover, the number of rail car types is reduced from 41 to 9. This smaller set of different types has additional indirect cost advantages. It enables a parking strategy of the rail cars on the storage tracks where each type is assigned to a separate track. Thus, a requested rail car of a specific type can be more easily accessed and provided.

In terms of future research it is worthwhile to analyze the gap between the presented sequential approach and a stochastic inventory model that directly accounts for the substitution possibilities. Due to the potentially large number of rail car types and substitution relations, as observed in our practical example, the development of fast solution procedures for such an inventory model is most likely required.

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Figure 2: Normal distribution approximation (Empirical - dashed, Normal - solid)