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Another look at the early-type eclipsing binary BF Aurigae

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Abstract. Since the question of the mass ratio of the early-type system BF Aur has not yet fully been clarified, we reanalyze existing B and V light curve data from the literature with special consideration to the photometric mass ratio, which we determine as $q=1.05\pm0.05$. The basic indeterminacy in $q_{\rm phot}$ resulting from the light curve shape, i.e. the impossibility to decide from photometry alone which star is in front at primary minimum, is overcome by considering the spectral line ratios. The finally adopted stellar parameters are consistent with available Strömgren indices for BF Aur. We confirm earlier conclusions that the more massive component is almost filling its Roche lobe. At present, there are no indications of mass transfer or period changes. The light curves, however, show some wavelength-dependent extra light and disturbances around phase 0.25 that might be related to stellar winds.

Key words: close binaries – eclipsing binaries

1. Introduction

BF Aur (=HD 32419=BD+41°1051; $\alpha_{1950}=05^{\rm h}01^{\rm m}33\overset{\rm s}{.}0$, $\delta_{1950}=+41°13'13''$, $P=1\overset{\rm d}{.}5832179$) was discovered as an eclipsing binary by Morgenroth (1935). The first photoelectric light curves in the yellow and blue spectral region were secured by Schneller (1961) who failed to get a consistent photometric solution with the rectifiable Russell–Merrill (1952) model. The light curve shape, however, indicated strongly deformed components of equal surface brightness and presumably also similar radii and masses, with relative radii $r_{\rm g}\approx r_{\rm k}\approx0.4$, probably forming a contact binary. The spectral classification, B5 V (Roman 1956), was found roughly consistent with normal mainsequence components of $\approx 5M_\odot$.

UBV light curves of moderate quality were obtained by Mannino et al. (1964). They also noted difficulties and inconsistencies with the Russell–Merrill model for the quasi-contact configuration encountered in BF Aur. The system was investigated spectroscopically at Asiago (Mammano et al. 1974) and claimed as an early-type contact system. However, subsequent analysis by Schneider et al. (1979; hereafter SDL) with the Wilson–Devinney (WD) program (using the more realistic Roche model), led to the conclusion that the system is in a semi-detached configuration, with the more massive component filling its Roche lobe. Such systems are unstable and susceptible to runaway mass transfer, and BF Aur is the earliest of the very few suspected systems, thus of special interest.

According to SDL, both components have evolved off the main-sequence, but the evolutionary history of the system is not clear: due to uncertainties in the spectroscopic orbit, SDL were not able to present a unique model; models with $q=0.83$ and $q=1.20$ were found to fit the observed light curve almost equally well, while both gave significantly better fits than the spectroscopic $q=1$, which lead to a detached configuration. SDL favored the $q=0.83$ solution as it seemed to be more consistent with the observational data, but recommended new spectroscopic observations to restrict the possible configurations as well as a new period study to test the predicted mass flow rate by observing the resulting period change.

In the analysis of SDL, the geometry had to be specified in advance and the question of the photometric mass ratio is not fully explored. Also, the treatment of the spectroscopic data is unsatisfactory and the line ratios have been paid no attention to. Thus we think that their results are of limited significance but certainly interesting enough to warrant a closer look at the system. A new method to optimize parameters in solutions of eclipsing binary light curves, the simplex algorithm, has recently been introduced by Kallrath & Linnell (1987; hereafter KL) and been reviewed in Kallrath (1992). The application of the software package LCCTRL (developed as part of JK's diploma thesis, Kallrath 1987) to BF Aur continues the work started in KL and a paper by Linnell & Kallrath (1987) with the analysis of the early-type detached close binary MR Cyg; here, the simplex algorithm is applied to a geometrically different situation.

In this paper, we present a photometric reanalysis of the observations by Mannino et al. (1964) with the WD model using the simplex optimization, and discuss the evolutionary status of BF Aur anew in the light of the available spectroscopic and photometric evidence.

2. The light curve analysis

2.1. Photometric solution

To solve the light curves, we used the software package LCCTRL as described by KL or Kallrath (1987). Calculation of theoretical light curves is based on the 1978 version of the WD code. The least-squares problem of parameter estimation is treated in the usual way: Given a physical model and a parameter vector $\boldsymbol{x}=(x_1, x_2, \ldots, x_m)$, we seek a solution in multidimensional parameter space minimizing the quadratic form $(\boldsymbol{d}, \boldsymbol{Wd})$ where $\boldsymbol{d}=\boldsymbol{o}-\boldsymbol{c}$ is the vector of light residuals (observed minus calculated intensities) and $W_{i\nu}=w_i\delta_{i\nu}$ (with $\delta_{i\nu}$ as the Kronecker

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Send offprint requests to: B.-C. Kämper symbol) specifies the assigned individual weights of the observations. As a measure for the quality of fit we take

$$\sigma_{\rm fit} := f(\boldsymbol{x}) = \left\{ \left(\frac{n}{n-m} \Big/ \sum_{\nu=1}^{n} w_\nu \right) \sum_{\nu=1}^{n} w_\nu \, [d_\nu(\boldsymbol{x})]^2 \right\}^{1/2}, \tag{1}$$

since this quantity is normalized with respect to the number n of observed data points and number m of adjusted parameters. To minimize $\sigma_{\rm fit}$, we use both the simplex algorithm as outlined in KL and the method of differential corrections already implemented in the WD code.

In our analysis performed with LCCTRL we used the same photometric data as SDL; we avoided the somewhat questionable practice of forming normal points, and included all observations (about 470 individual data points in each colour) when analyzing the data with the simplex algorithm. The weights were chosen proportional to $1/l$, as appropriate for photon statistics ($b=1/2$ in the nomenclature of Linnell & Proctor 1970).

All calculations were performed in mode 2 (cf. Wilson 1991), i.e. L_2 is coupled to T_2 through the Planck function, while no geometrical constraints are imposed. In LCCTRL *all* configurations possible in the Roche model (detached, semi-detached, contact, over-contact) may be realized within mode 2. That means, if one or both stars overfill their Roche lobe, the corresponding lobe filling constraint is automatically applied. We fixed the temperature of the primary at $T_1=15400$ K estimated from its spectral type B5 V (Popper 1980).

For the linearized limb darkening coefficients, we adopted the values given by Wade & Rucinski (1985), which are based on the model-atmosphere grid of Kurucz (1979); for BF Aur, we got $x_1(B)=x_2(B)=0.36$, $x_1(V)=x_2(V)=0.31$. The strong correlation of the limb darkening coefficients with other light curve parameters and its negative influence on the numerical properties of the light curve problem is a well-known problem (see e.g. Wilson & Devinney 1971 or Twigg & Rafert 1980). Only under exceptional circumstances (viz. total eclipses) it will be possible to extract meaningful information about limb darkening coefficients. In view of these difficulties the wisest is perhaps to trust the coefficients derived from the best available model atmospheres.

The albedos were fixed at the values appropriate for radiative atmospheres, $A_1=A_2=1$, corresponding to full reradiation. Gravity darkening exponents $g_1=g_2=1$ were chosen, corresponding to von Zeipel's law.

First we analyzed the B and V curves separately. The results are given in Table 1. Due to an asymmetry in the V curve, $\sigma_{\rm fit}(B)$ is less than $\sigma_{\rm fit}(V)$. However, there are only slight differences between the separate solution of the B light curve and the four different V curve test runs.

Since the obtained solutions were consistent a simultaneous analysis of the B and V curve seemed warranted. As a consequence of the decreased number of free parameters in a simultaneous analysis, the $\sigma_{\rm fit}$ of the simultaneous solutions, given in Table 2, is larger. Columns 1–6 give an impression how sensitive the quality of the fit depends on the parameters. Column 6 gives the finally adopted "second solution" with inverted mass ratio (see discussion below). The solution shown in column 2 is typical for the behaviour of a simplex which has contracted too fast; it is discussed further below. In order to distinguish and single out such false "solutions" (local minima in the $\sigma_{\rm fit}$ hypersurface) one should take into account the other values of σ as well as external evidence.

For completeness, we give in Table 3 those values which can be derived from the finally adopted solution $\boldsymbol{x}_6$ (surface S_k, volumes V_k, mean radii $\bar{r}_k$). Figure 2 shows the light curves and residuals for this solution.

Additional light curves based on various parameter combinations that occurred during the optimization procedure (not shown in this paper) demonstrated that the parameter solution is poorly determined. The shape of the light curve and the quality of the fit was very similar in all cases although the parameters had quite different values.

2.2. *The ambiguity in the photometric mass ratio*

Figure 1 is used to support the discussion of the second solution near $q=1.20$. The BF Aur light curves are characterized by two minima of nearly equal depth. The physical reason for this phenomenon is the small temperature difference of both components. Furthermore, the shape of both minima is nearly identical. The following statements not only hold for BF Aur, but for all systems with the properties given above: From photometry alone, it is not possible (or at least very difficult) to assign the components to the primary or secondary minimum, i.e. to decide whether primary minimum is a transit or an occultation. Nevertheless, relative system parameters may be derived. Let $\boldsymbol{x}_*$ be a solution of the system; then $\boldsymbol{x}'_*$ also is a solution when a transformation $\hat{T}$ is applied that simply describes the exchange of both components. In the coordinate frame of the primary (defined to be the component eclipsed at phase zero), $\hat{T}$ is formally expressed as

$$q' \to \frac{1}{q}, \qquad \Omega'_1 = \frac{1}{q}\Omega_2 + \frac{1}{2}\left(1 - \frac{1}{q}\right), \qquad \Omega'_2 = \frac{1}{q}\Omega_1 + \frac{1}{2}\left(1 - \frac{1}{q}\right). \tag{2}$$

The photospheric parameters T, L, x, g and A are only exchanged.

The transformed light curve $L'_{\rm cal} := L_{\rm cal}(\boldsymbol{x}'_*)$ and the original light curve $L_{\rm cal}(\boldsymbol{x}_*)$ are related by

$$l'_c(\Psi) = l_c(\Psi + 0.5). \tag{3}$$

Light curves with the properties given above approximately obey:

$$l'_c(\Psi) \cong l_c(\Psi). \tag{4}$$

Table 4 gives the solution $\boldsymbol{x}_*$ and the related transformed solution $\boldsymbol{x}'_*$. Figure 1 shows the difference of the light curves generated with these parameter values. A good fit is easily achieved if one adjusts the temperature of the secondary, T_2. The remaining differences are below $0^{\rm m}\!.01$, and the achieved light curve fits are practically undistinguishable from those with the inverse mass ratio. So we conclude that it is impossible to make a decision about the mass ratio (respectively the transit/occultation question) from the photometry alone.

2.3. *Consideration of the spectral line ratios*

From the light ratios we got in our solutions (in the B colour, 0.66 for the transit solution near $q=0.85$, 1.41 for the occultation solution near $q=1.15$), we expected that the matter could be settled by simple inspection of the relative strengths of the spectrum lines. (As both components have almost equal temperatures and similar surface gravities the line ratios should compare well

Table 1. Photometric single-colour solutions for BF Aur. k gives the number of iterations, $\Delta\varphi$ the phase range of the light curve used for analysis, $\sigma := 10^5\ \sigma_{\rm fit}$ the standard deviation in light units

no.	U1	B1	V1	B1/DC	U2	B2	V2
$\Delta\varphi$	2nd	2nd	2nd	2nd	2nd	2nd	2nd
i	84.93	84.68	84.71	84.72 ±.10	85.01	84.89	84.83
q	1.05	1.05	1.05	1.05	1.15	1.15	1.15
T_1,p	16700	16700	16700	16700	16700	16700	16700
T_2,p	16805	16872	16944	16781 ±58	16623	16656	16606
T_1,m	15988	15990	15983	15980	15918	15937	15930
T_1,m	15901	15953	16016	15950	15748	15822	15726
$\Delta T/T_1$	0.005	0.002	-.002	0.002 ±.003	0.011	0.007	0.013
Ω_1	4.073	4.075	4.067	4.062 ±.018	4.142	4.167	4.160
Ω_2	3.846	3.835	3.832	3.832 ±.010	4.031	4.082	4.026
F_1	94 %	94 %	94 %	94 %	96 %	96 %	96 %
F_2	99.6%	99.9%	100%	100%	99 %	98 %	99.1%
r_1	0.341	0.341	0.342	0.342	0.346	0.343	0.344
r_2	0.382	0.384	0.384	0.384	0.386	0.379	0.387
$k=r_2/r_1$	1.119	1.126	1.124	1.121	1.116	1.105	1.126
L_1	0.450	0.445	0.443	0.447 ±.005	0.454	0.455	0.448
J_2/J_1	1.016	1.024	1.029	1.023	0.988	0.994	0.989
$\overline{J_2/J_1}$	0.972	0.981	0.991	0.981	0.964	0.979	0.968
x	.38	.36	.31	.36	.38	.36	.31
σ (in $\Delta\varphi$)	1421	1120	1093	925	1144	937	1044
σ (total)	1439	913	1126	849	1195	830	1086

no.	B3	V3	V4	B5/SD	B6/SD	B7/SD	B8/SD
$\Delta\varphi$	1st	1st	1st	1st	1st	1st	1st
i	84.78	85.15	84.82	84.63	84.36	85.12	85.24
q	1.15	1.15	1.35	1.00	1.045	1.10	1.147
T_1,p	16700	16700	16700	16700	16700	16700	16700
T_2,p	16703	16513	16156	16946	16833	16991	16888
T_1,m	15955	15957	15886	16047	16006	16052	16017
T_1,m	15849	15719	15402	16020	15913	16061	15963
$\Delta T/T_1$	0.007	0.015	0.030	0.002	0.006	-.001	0.003
Ω_1	4.192	4.196	4.422	4.077	4.091	4.253	4.277
Ω_2	4.061	4.128	4.497	3.75	3.823	3.911	3.985
F_1	95 %	95 %	97 %	98 %	93 %	91 %	93 %
F_2	98 %	97 %	95 %	100%	100%	100%	100%
r_1	0.339	0.339	0.337	0.334	0.338	0.326	0.329
r_2	0.382	0.372	0.381	0.380	0.384	0.388	0.392
$k=r_2/r_1$	1.125	1.099	1.130	1.137	1.135	1.192	1.193
L_1	0.446	0.461	0.454	0.440	0.443	0.415	0.417
J_2/J_1	1.000	0.978	0.936	1.034	1.018	1.040	1.026
$\overline{J_2/J_1}$	0.979	0.968	0.953	0.979	0.971	0.985	0.977
x	.36	.31	.31	.36	.36	.36	.36
σ (in $\Delta\varphi$)	717	1333	1292		760	754	768
σ (total)	928	1076	1025	960	970	952	952

to the luminosity ratio.) Popper (1981) has reproduced microphotometer tracings of the spectra of 26 OB eclipsing binaries in the wavelength range 430–450 nm, including BF Aur (cf. Fig. 7 of his paper). The spectrogram illustrated corresponds to phase 0.70, i.e. the component eclipsed at primary minimum is receding. In the tracing the lines of both components are clearly present, the redshifted component being the weaker one. From Hγ and He I 4388, we estimate a line ratio of roughly 1.3 ± 0.1, in agreement with the statement made already by Mammano et al. (1974) (but ignored by later investigators) that the spectroscopic component 2 is that eclipsed at primary minimum. Thus the first solution is in clear contradiction with spectrographic evidence and we have to adopt the second solution near $q = 1.15$.

We want to stress, however, that the basic configuration remains the same: in both cases the more massive component, almost filling its critical Roche lobe, is also the more luminous

Table 2. Photometric solutions for BF Aur: Results of the simultaneous analysis in U, B and V. k gives the number of iterations, $\sigma := 10^5\ \sigma_{\mathrm{fit}}$ the standard deviation in light units. Column 1 shows the results of a simplex run with the initial simplex chosen according to SDL's first solution (rejected afterwards). The transit solution 2 was also rejected and is commented in the text. Columns 3–7 give different solutions with mass ratio larger than 1

no.	S1	S2	1	2	3	4	5	6	7	
type			"S1"	"Tr"	"S2"	"S2"	"S2"	"S2"	"S2"/DC	
i	85.14	84.89	85.13	84.29	84.96	84.79	84.71	84.78	84.68	± .05
q	0.831	1.200	0.830	1.139	1.223	1.05	1.10	1.05	1.05	
T_1,p	15600	15600	15400	15400	15600	16700	16700	16700	16700	
T_2,p	14900	15609	14718	13998	15756	16849	16688	16850	16851	± 27
T_1,m						15977	15936	15974	15978	
T_2,m						15945	15813	15946	15939	
$\Delta T/T_1$						0.002	0.008	0.002	0.002	±.002
Ω_1	3.470	4.198	3.507	3.974	4.345	4.059	4.085	4.052	4.059	±.009
Ω_2	3.591	4.068	3.645	4.571	4.131	3.851	3.953	3.849	3.841	±.006
F_1			99 %	100 %	94 %	94 %	96 %	94 %	94 %	
F_2			95 %	85 %	99.4%	99.5%	99 %	99.5%	99.7%	±0.2%
r_1			0.389	0.368	0.330	0.343	0.346	0.344	0.343	
r_2			0.334	0.320	0.394	0.381	0.382	0.381	0.383	
$k=r_2/r_1$			0.859	0.868	1.193	1.111	1.102	1.109	1.116	
$L_1(U)$					-.---	0.451	0.458	0.452	0.450	±.003
$L_1(B)$			0.598	0.611	0.415	0.450	0.457	0.451	0.449	±.003
$L_1(V)$			0.596	0.605	0.415	0.450	0.456	0.451	0.448	±.003
J_2/J_1 (U)					-.---	1.024	0.998	1.024	1.024	
J_2/J_1 (B)					1.024	1.020	0.998	1.021	1.020	
J_2/J_1 (V)					1.020	1.018	0.999	1.018	1.018	
$\overline{J_2/J_1}$ (U)					-.---	0.982	0.971	0.981	0.979	
$\overline{J_2/J_1}$ (B)					0.987	0.984	0.975	0.985	0.982	
$\overline{J_2/J_1}$ (V)					0.988	0.986	0.978	0.986	0.984	
x (U)					-.---	.38	.38	.38	.38	
x (B)	.31	.31	.31*	.31*	.36	.36	.36	.36	.36	
x (V)	.26	.26	.31	.31	.31	.31	.31	.31	.30	
$\sigma(UBV)$			1149	1543	1155	1276	1044		1090	(842)
$\sigma(U)$					----	1220	1154	1140		
$\sigma(B)$	1052	1060	1150		914	788	787	940		
$\sigma(V)$	1260	1268	1310		1213	1111	1057	1290		

*) the limb darkening in B was inadvertently set equal to that in V for the first solutions.

and larger one. The main difference to the first solution lies in the relative temperature difference which has been lowered (as expected). Note that column 2 in Table 2 is *not* a solution close to S_2 as superficial inspection might suggest. It was the result of an initial simplex near S_2 but with the luminosities reversed (inadvertently); in this case the simplex contracted too fast and dropped into a local minimum of the multidimensional parameter surface corresponding to a configuration with the less massive component filling its critical Roche lobe, while being the hotter, more luminous and larger one (thus contradicting a normal main-sequence interpretation). However, the light curve fit is bad and this solution can be excluded (even though the transformation (2) would lead to another solution near $q=0.85$ with $L_2/L_1>1$).

2.4. *A parameter study for BF Aur*

Experience has shown that the mass ratio, although it may in principle be obtained for close binary systems from light curve synthesis, is often a quite weakly determined parameter, especially in near-contact binaries (see e.g. the discussions in Breinhorst et al. 1989; Kałużny & Semeniuk 1984). It is true (as the referee pointed out) that a semi-detached solution constraint may often give a quite definitive photometric mass ratio, being based more on the size of the lobe filling star than its distortion; however, can we be sure that the system is really semi-detached? Moreover, a photometric mass ratio may be easily biased by photometric distortions outside eclipse, as they are present in BF Aur. For a configuration like BF Aur, there is no substitute for a reliable spectroscopic mass ratio; however, as we don't have

Table 3. Full parameter set describing the adopted light curve solution for BF Aur, together with estimated uncertainties of the main parameters

$q = \mathcal{M}_1/\mathcal{M}_2 = 1.05 \pm 0.05$	$\Omega_c = 3.831$	$\Omega_1 = 4.052$	$\Omega_2 = 3.849$
r_1(pole) = 0.327	r_2(pole) = 0.358	$F_1 = 91.6\%$	$F_2 = 99.2\%$
r_1(point) = 0.385	r_2(point) = 0.472	$S_1 = 1.488$	$S_2 = 1.838$
r_1(side) = 0.340	r_2(side) = 0.376	$V_1 = 0.170$	$V_2 = 0.232$
r_1(back) = 0.327	r_2(back) = 0.358	$\bar{r}_1 = 0.344 \pm 2$	$\bar{r}_2 = 0.381 \pm 2$
$i = 84°.8 \pm 0°.5$		$k = \bar{r}_2/\bar{r}_1 = 1.11 \pm 0.04$	
(L_2/L_1)U = 1.214	$(J_2/J_1)_{\rm pole} = 1.024$	$(\overline{J_2/J_1})$U = 0.981	
(L_2/L_1)B = 1.216	$(J_2/J_1)_{\rm pole} = 1.021$	$(\overline{J_2/J_1})$B = 0.985	
(L_2/L_1)V = 1.218	$(J_2/J_1)_{\rm pole} = 1.018$	$(\overline{J_2/J_1})$V = 0.986	
$\ell_2/\ell_1 = 1.226$ $(\Delta M_{\rm bol} = 0.22)$		$\Delta T = (T_1 - T_2)/T_1 = 0.002 \pm .005$	

Ω_c = critical Roche potential, $\Omega_{1,2}$ = dimensionless surface potentials,

$F_{1,2}$ = filling factors, $S_{1,2}$ = (norm.) areas, $V_{1,2}$ = (norm.) volumes,

$\bar{r}_{1,2}$ = volume radii, i = inclination, q = mass ratio, k = ratio of radii

L_2/L_1 = ratio of (monochromatic) luminosites, integrated over the whole surface of the stars (in absence of the reflection effect),

$(J_2/J_1)_{\rm pole}$ = ratio of normal emergent intensities at the pole,

$\overline{J_2/J_1}$ = ratio of mean surface brightnesses.

that, we will do the next best thing, viz. (1) try to evaluate and avoid as far as possible the detrimental effect of light curve distortions on the solution, and (2) perform a careful parameter study with the aim of getting not just the best formal photometric fit but the most overall consistent parameter set.

First we have to decide which phase ranges are better excluded because of distortions not included in the model used for light curve synthesis. The basic light curve distortion is obviously the asymmetry between the first and second half of the light curve, apparent as a non-vanishing magnitude difference Δm between Max I and Max II. After some tests we decided to use only the second half of the light curve ($\varphi = 0.5 \ldots 1.0$), on the basis that only for this phase range we find consistent parameter sets in U, B and V (the other half gives a higher q for the V curve and a lower value for the U curve). This is also obvious from a simple morphological consideration: if we have stars with almost equal temperatures, as indicated by the depths of primary and secondary minimum, we should expect very similar light curve shapes and almost equal depths in all colours. Superposition of all three light curves shows that this is fulfilled only for the second half.

For a grid study in q we selected the B curve because it is the most symmetrical of the three. The mass ratio was varied between 0.9 and 1.8 in steps of 0.1, iterating at each fixed value of q to convergence. The resulting photometric solutions are given in Table 5. Some important parameters of the obtained grid of light curve solutions are shown in Figs. 4–6.

To our surprise we could not verify SDL's detached solution for $q = 1$. To the contrary, all solutions with $q < 1.05$ converged to a semi-detached configuration (SD1), while for $q > 1.05$ the primary underfills its Roche lobe to an increasing degree albeit it stays quite close to contact (Fig. 6). This shows how unreliable and misleading can be a result achieved by performing just a few differential corrections iterations near a preliminary solution. Our best fit solution was achieved at $q = 1.15$ (cf. Fig. 4) and it is almost semi-detached. Note that SDL used the WD code in mode 5 and thus were forced to adopt a truly semi-detached configuration. They point out that one should then expect BF Aur to be in the rapid phase of mass transfer from the more massive to the less massive component (proceeding on a thermal time scale) and thus to see a corresponding period decrease. This, however, is not observed; the period has remained practically

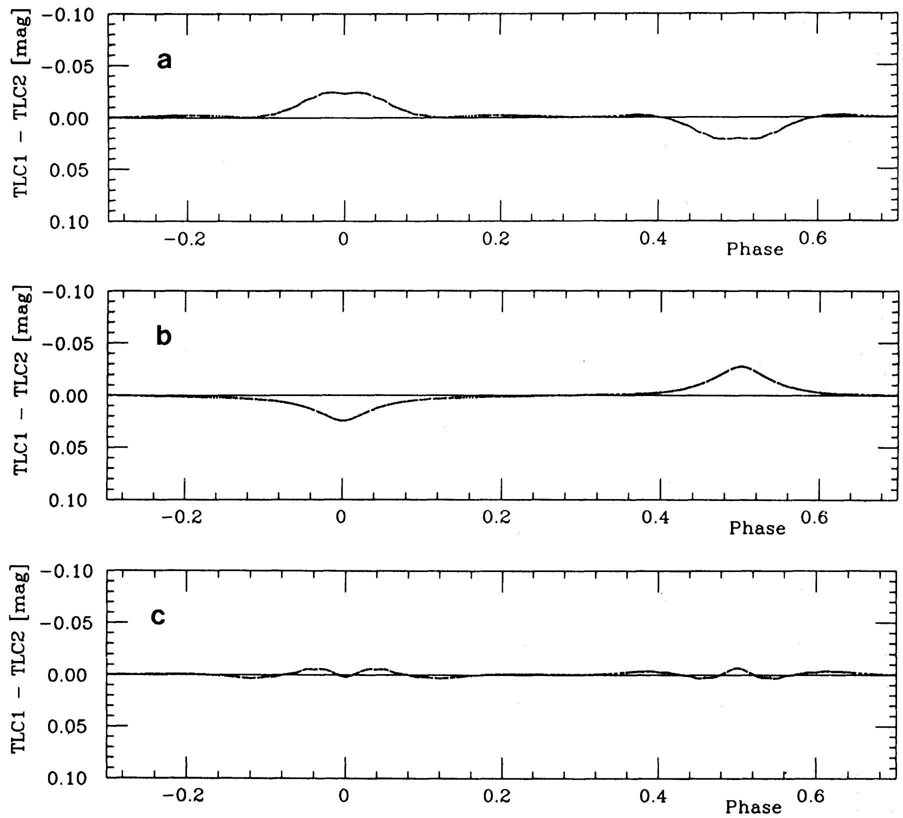

Fig. 1. a Difference between best fit BF Aur B light curve at $q=1.15$ and another one calculated for inverted geometry ($q=0.87$), using the transformed parameter set (cf. Table 4). **b** shows the effect of adjusting T_2 until **c** a good fit is achieved again

constant over more than 70 years of observational records (see Fig. 8). One reason for this may be that the period change due to mass transfer is proportional to the divergence of q from 1, which probably is much smaller than thought before; it may be estimated that if only 5–10% of the material lost by the primary is lost from the system (non-conservative case), a period decrease might be compensated or even turned into a period increase. On the other hand, our light curve analysis suggests that BF Aur is still barely under-filling its Roche lobe so that mass transfer has not yet fully developed.

Now we have several constraints in regard to the mass ratio: a rather poor spectroscopy which gives $q=1.04\pm0.03$ (probably rather a lower limit, see discussion below), the semi-detached solution constraint, which leads to a rather sharply determined lower limit of $q=1.05$ for the photometric mass ratio, and finally the photometric best fit value, $q=1.15$, with an accuracy probably not better than ±0.1. On the other hand there is the mass–luminosity relation which we expect to be fulfilled for main-sequence components of close binary systems as long as mass transfer has not yet influenced their evolution. From the mass–luminosity relation, we would expect a larger disparity in bolometric magnitude than the observed $\Delta M_{\rm bol}=10\log(T_2/T_1)+5\log k\approx0\overset{\rm m}{.}2$, if q were near 1.2, the value which formally gives the best fit to the light curve. Taking $L\approx M^4$ (Smith 1983), we should have $\Delta M_{\rm bol}\approx10\log q\approx0\overset{\rm m}{.}8\pm0\overset{\rm m}{.}2$. A solution compatible with the mass–luminosity relation should fulfill $q\approx\sqrt{k}$ for equal temperatures. Since $k\approx1.12\pm0.01$ for all reasonable solutions, this indicates again a q near 1.06. Figure 5 shows that the bolometric magnitude difference between the binary components as predicted from the light curve solutions is a flat function of q (which means that the observed spectral line ratios can only be used to solve the transit/occultation question but not to discriminate between different mass ratios) while the mass–luminosity relation is a steep function of q. The intersection occurs at $q\cong1.06$.

2.5. *The influence of the reflection effect on the solution*

Kitamura & Yamasaki (1984) and, recently, Wilson (1990) considered the effect of "multiple reflections" on the modeling of the reflection effect in the context of the Roche model. Both consider only the bolometric heating effect caused by the mutual irradiation of the stars atmospheres, not the rather more complex problem of the redistribution of the reflected energy over wavelength. [In the Wilson paper, monochromatic light curves are calculated based on the usual simplified assumption that the heated photosphere locally radiates like an unirradiated (black body or model) atmosphere of the same effective temperature.] Wilson gives simulations for BF Aur as an example for a system in which multiple reflection is appreciable; his Table 2 shows that the effect amounts to 15% (for the primary) resp. 20% (for the secondary) of the first order reflection when averaged over the "reflection cap".

Kitamura & Yamasaki (1984) computed the reflection effect by light curve synthesis for a set of model binaries with equal components which allowed them to measure the reflection effect (of first and higher orders) by its contribution $C_{\rm ref}(n)$ to the coefficient of the cos 2θ term in the light variation outside eclipse and compare it with the contribution $C_{\rm g}$ due to ellipticity and gravity darkening alone. Their model for [$r=0.7$, $u=0.4$, $w(\equiv{\rm A})=1$, $\alpha(\equiv{\rm g})=1$] may be taken as representative for a system like BF Aur. For this model, $C_{\rm ref}(2)$ amounts to 20% of

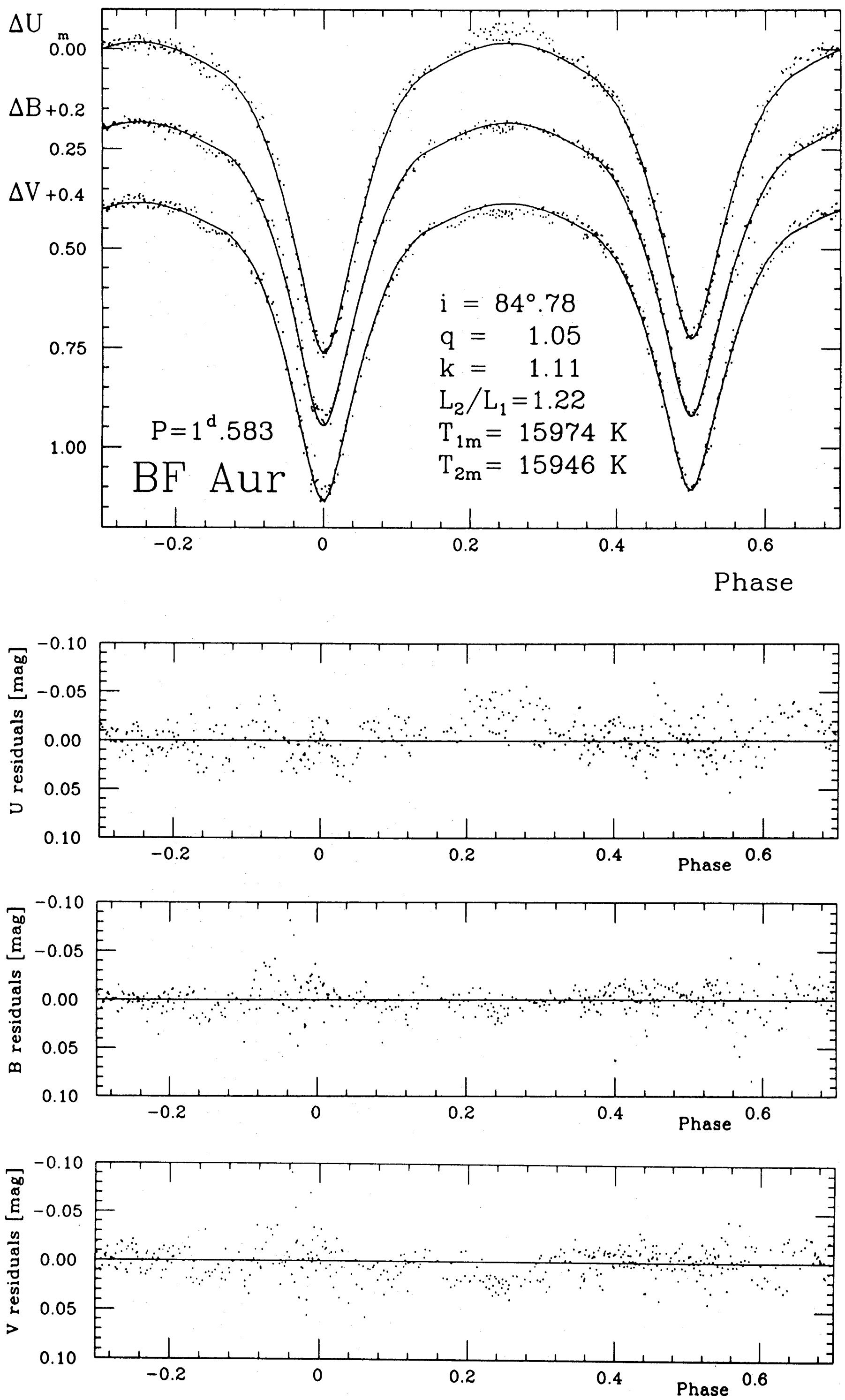

Fig. 2. BF Aur *U*, *B*, and *V* light curve fits and residuals, adopted simultaneous *UBV* solution at $q=1.05$ with LCCTRL and the simplex algorithm (solution 6 of Table 2). Note the differing heights of Max I and Max II in the *U* and *V* colours

$C_{\text{ref}}(1)$, in good agreement with the estimate we got from Wilson's (1990) paper. However, $C_{\text{ref}}(2)/C_g$ is only 1.5%, and the absolute values of the coefficients are small, $C_{\text{ref}}(1)\cong 0.011$, $C_{\text{ref}}(2)\cong 0.002$.

On the other side, the work of Vaz & Nordlund (1985) on the reflection effect in grey model atmospheres indicates that an irradiated atmosphere is "redder" than an unirradiated atmosphere of the same effective temperature. This implies, for small values of $hc/\lambda kT$, an "effective albedo" greater than 1. For BF Aur, we find that this effect could give another 20% increase. In total, the absolute magnitude of C_{ref} could rise by 0.004 compared to the simple reflection calculation. This shows that the influence on the results cannot be serious.

Qualitatively, underestimating the magnitude of C_{ref} (the sign of which is negative) means that the ellipticity effect remaining after the subtraction of the reflection contribution is greater than

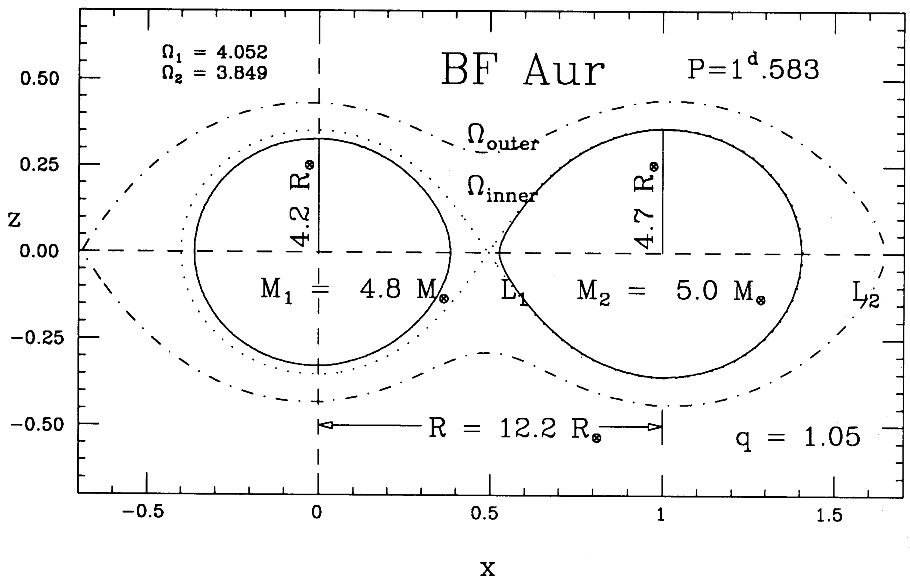

Fig. 3. A geometric model of BF Aur based on the simultaneous *UBV* solution at $q = 1.05$ and an assumed main-sequence mass for the primary component

Table 4. BF Aur (*B* light curve) – Solution at $q = 1.15$ and transformed solution with inverted geometry. Fixed parameters were $i = 84.7°$, $g_1 = g_2 = A_1 = A_2 = 1$, $x_1 = x_2 = 0.36$. Only radiative parameters were allowed to readjust

	q	Ω_1	Ω_2	T_{1p}	T_{2p}	T_{1m}	T_{2m}	L_1	L_2/L_1	$\overline{J_2/J_1}$	σ_{fit}
x_*	1.15	4.150	4.057	16700	16677	15924	15821	0.453	1.207	0.980	0.0105
x_*'	0.87	3.593	3.674	16677	16224	15821	15470	0.564	0.773	0.952	0.0106

Table 5. Photometric *B* colour solutions for BF Aur, for a fixed grid of mass ratios q. The first half of the light curve was used for analysis. $\sigma := 10^5\ \sigma_{fit}$ is the standard deviation in light units

no.	B#1	B#2	B#3	B#4	B#5	B#6	B#7	B#8	B#9
i	85.00	84.65	84.63	84.77	84.83	84.78	84.94	85.27	85.49
q	0.90	0.95	1.00	1.05	1.10	1.15	1.20	1.30	1.40
T_1,p	16700	16700	16700	16700	16700	16700	16700	16700	16700
T_2,p	17140	17054	16946	16769	16761	16703	16520	16276	16151
T_1,m	16108	16074	16047	16059	15975	15955	15926	15881	15844
T_1,m	16204	16122	16020	15873	15886	15849	15688	15506	15429
$\Delta T/T_1$	-.006	-.003	0.002	0.008	0.006	0.007	0.015	0.024	0.026
Ω_1	4.005	4.034	4.076	4.103	4.137	4.192	4.234	4.334	4.445
Ω_2	3.586	3.668	3.750	3.853	3.958	4.061	4.158	4.394	4.631
F_1	88 %	90 %	91 %	93 %	94 %	95 %	96 %	97 %	98 %
F_2	100 %	100 %	100 %	99.4%	99 %	98 %	98 %	96 %	94 %
r_1	0.330	0.333	0.334	0.337	0.340	0.339	0.341	0.342	0.341
r_2	0.370	0.375	0.380	0.381	0.381	0.382	0.383	0.380	0.378
$k = r_2/r_1$	1.122	1.128	1.137	1.128	1.122	1.125	1.124	1.114	1.109
L_1	0.442	0.441	0.440	0.447	0.448	0.447	0.451	0.460	0.463
J_2/J_1	1.061	1.049	1.034	1.009	1.008	1.000	0.976	0.943	0.926
$\overline{J_2/J_1}$	0.994	0.989	0.979	0.968	0.979	0.979	0.962	0.948	0.947
σ (in $\Delta\varphi$)	860	784	760	737	729	717	724	789	844
σ (total)	1040	985	950	941	939	928	939	958	1030

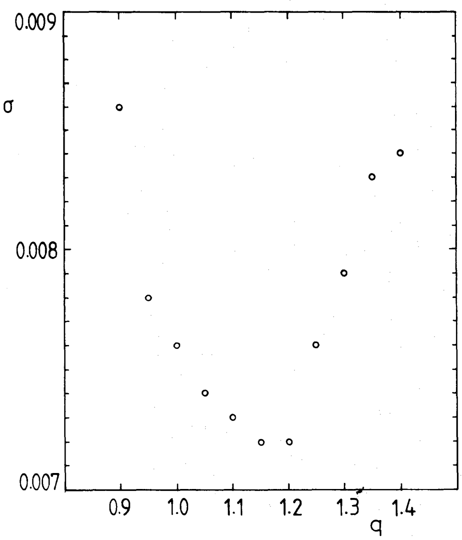

Fig. 4. Grid study of the BF Aur B light curve: Quality of fit as function of mass ratio

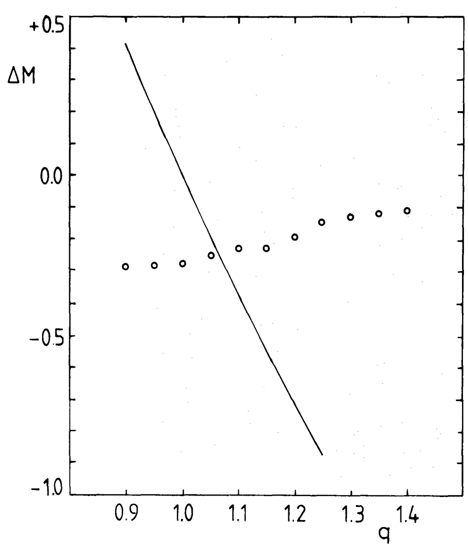

Fig. 5. Grid study of the BF Aur B light curve: $\Delta M_{\rm bol}$ as derived from the photometric solution (open circles) compared with the mass–luminosity relation (solid line)

assumed. Correcting this would shift the photometric mass ratio in the direction to smaller values and thus further reduce the discrepancy to the spectroscopic value. A simple increase of the albedos by 40% should suffice to test this supposition. We performed the test in two ways: first we generated *synthetic* light curves for an albedo of 1.4 and a mass ratio of 1.05. To these we applied the simplex algorithm and looked for a solution with the albedo fixed at $A=1.0$; i, T_1, L_1, Ω_1, and q were allowed to readjust. The simplex converged to a solution with mass ratio $q \cong 1.1$. As a second test, we reanalyzed the *observed* light curves with an albedo of 1.4. Several test runs of the program with

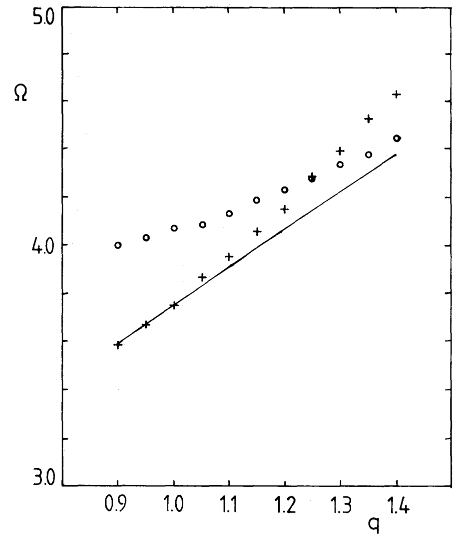

Fig. 6. Grid study of the BF Aur B light curve: Roche potentials of both system components (+primary, o secondary) as compared with the critical Roche potential

$q=1.05$, 1.1 and 1.15 showed that the pronounced χ^2 minimum at $q=1.15$ has disappeared and the goodness of fit is now sensibly equal at $q=1.05$ and 1.1. Thus, the discrepancy between the spectroscopic and photometric mass ratio is no longer significant. From the considerations in Sect. 2.5, we will adopt the most probable value, $q=1.05$. Table 6 compares the solutions found at this mass ratio with $A=1.0$ and 1.4. The influence of A on the other parameters is barely significant.

2.6. *Additional information from Strömgren indices*

Strömgren indices for BF Aur measured at five different phases (Table 7) are available from the survey of Hilditch & Hill (1975). Because both components have almost equal temperature and similar surface gravities, their colours will be similar and we may take the measured indices as representative for the mean component, after appropriate correction for interstellar extinction. The measured indices may be dereddened using Crawford's (1978) intrinsic colour relations for B-type stars. Taking the slope of the reddening line in the $(u-b)$ vs. $(b-y)$ diagram to be 1.5, one gets a colour excess $E(b-y)=0.154\pm0.01$, and thereby $(b-y)_0=-0.071$, $(u-b)_0=0.45$, $c_0=0.38$, $m_0=0.106$. From the colour excess one can estimate the total visual absorption $A_V \simeq 4.3\ E(b-y)=0.66$. This gives the combined visual magnitude corrected for interstellar extinction as $V_0=8.14$, and for the mean component we get $V_0=8.89$. From Moon's (1984) empirical calibration of the intrinsic colour index $(b-y)_0$ in terms of the visual surface brightness parameter F_V, defined as $F_V=4.2207-0.1V_0-0.5\log\phi_{1\rm d}$ (Barnes & Evans 1976), we then derive $F_V=4.080\pm0.03$, $\phi_{1\rm d}=0''.032\pm0.001$.

Various temperature calibrations using Strömgren indices lead to closely concordant results. For example, the (c_0, T_e)-calibration of Davis & Shobbrook (1977) for luminosity class V–III, in conjunction with Code et al.'s (1976) $(T_e$, BC)-relation gives

$$T_e \approx 16\,000 \pm 500\ \mathrm{K}, \qquad \mathrm{BC} \approx -1.52 \pm 0.10,$$

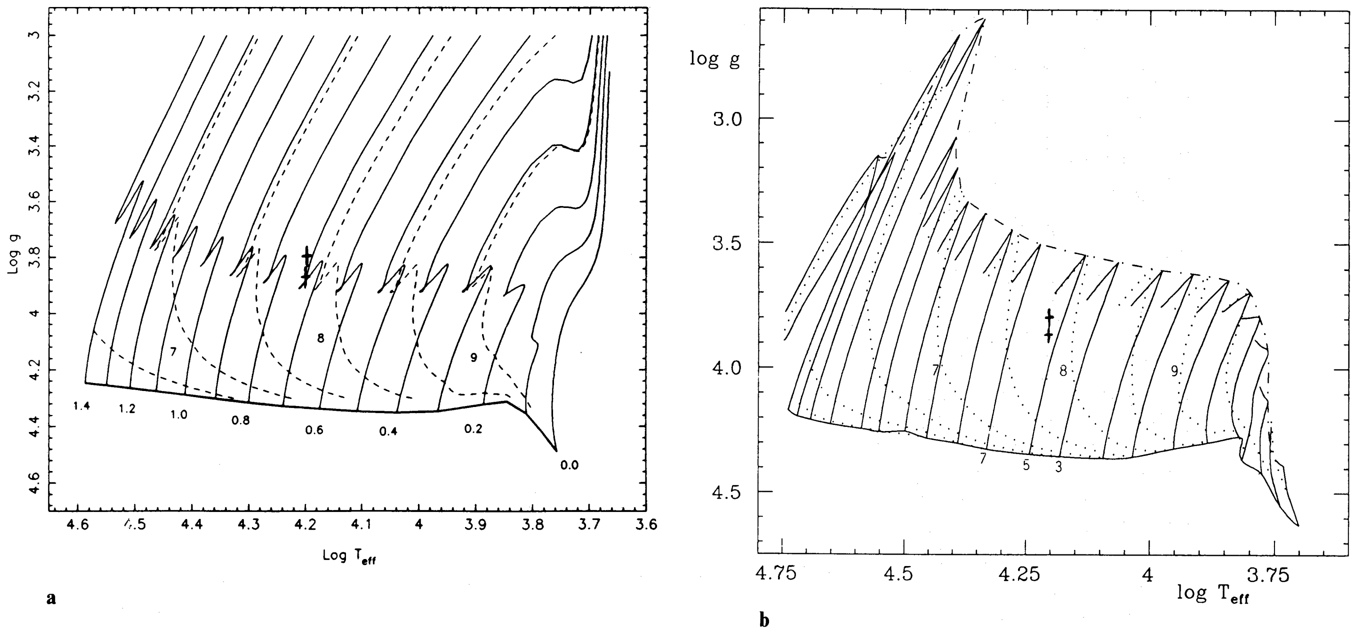

Fig. 7a and b. Position of both components (+primary, o secondary) of the BF Aur system in the evolution sensitive log g–log T_e plane. The lower solid line denotes the Zero Age Main Sequence, the upper dashed line the Terminal Age Main Sequence, taken from the models of **a** Claret & Giménez (1989), and **b** Maeder & Meynet (1989), respectively. Chemical composition is $(X, Z) = (0.70, 0.02)$. Evolutionary tracks are plotted as solid lines, isochrones as dotted lines

Table 6. Comparison of solutions with $A = 1.0$ and $A = 1.4$ (both at $q = 1.05$)

A	i	T_1,p	T_2,p	Ω_1	Ω_2	r_1	r_2	L_1	σ_{fit}
1.0	84.68	16700	16851	4.059	3.841	0.343	0.383	0.450	0.0117
1.4	84.44	16700	16840	4.054	3.838	0.344	0.383	0.450	0.0117

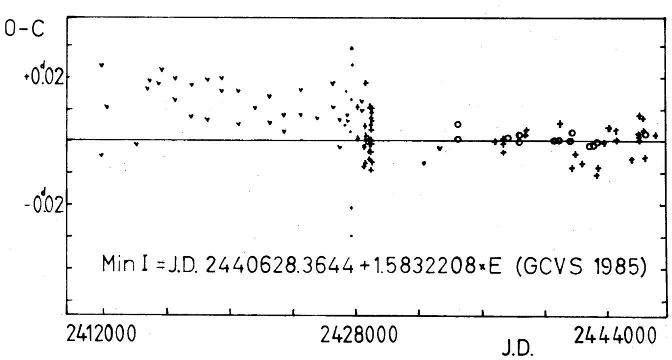

Fig. 8. O–C diagram of BF Aur (1891–1985) against the linear elements of the GCVS 1985. "‸" and dots represent epochs of faint light on sky patrol plates and photographic minima, respectively, crosses denote visual observations, filled circles photoelectric minima (courtesy D. Lichtenknecker (1989), based on the BAV eclipsing binary star files)

while Jakobsen's (1985a) improved empirical ($[u-b], T_e$)- and BC(T_e)-calibrations lead to

$T_e \approx 16\,100 \pm 500$ K, $\quad$ BC $\approx -1.40 \pm 0.10$.

From interpolation in the theoretical grids of Lester et al. (1986) we get

$T_e \approx 15\,900 \pm 500$ K, $\quad$ $\log g \approx 3.92 \pm 0.15$.

Finally, a photometric spectral type may be derived according to the calibration of Jakobsen (1985b) from its position in the $[m_1]$–$[c_1]$ plane, where it occupies the locus of an evolved B4-5 V star.

3. Discussion

Due to a more reliable photometric analysis, the discrepancy between the mass ratio of unity derived by Mammano et al. (1974) from their spectrographic orbit and the photometric mass ratio as found by SDL has been alleviated considerably, and the remaining difference is in magnitude and sense what we have to expect considering possible systematic error sources. Mammano et al. (1974) included Balmer lines in their measurements which are always severely blended and lead to a significant reduction in the measured radial velocity amplitudes (cf. Andersen et al. 1980). Presumably the weaker component will be most affected thus explaining the reduced q. The large difference in the systemic velocity, computed from separate solutions for the two components, is also clear evidence for systematic error due to strongly

Table 7. Strömgren indices of BF Aur, taken from the survey of Hilditch and Hill (1975). Reddening-free bracket indices were computed as $[c_1]=c_1-0.20(b-y)$, $[m_1]=m_1+0.32(b-y)$, $[u-b]=(u-b)-1.56$ $(b-y)$

Phase	(b-y)	c_1	m_1	[c_1]	[m_1]	[u-b]	V
0.8311*	0.092*	0.386*	0.030*	0.368*	0.059*	0.486*	8.79
0.0896	0.079	0.414	0.055	0.398	0.080	0.559	8.98*
0.1639	0.077	0.388	0.067	0.373	0.092	0.556	8.81
0.3492	0.091	0.434	0.046	0.416	0.075	0.566	8.80
0.4318	0.084	0.398	0.058	0.381	0.085	0.551	8.99*
mean	0.083	0.409	0.057	0.392	0.083	0.558	8.80
sdev	0.006	0.020	0.009	0.019	0.007	0.006	0.01

*) Measurements discarded in forming mean values.

Table 8. Spectroscopic orbits computed from the radial velocities given by Mammano et al. (1974), for different phase intervals around quadrature

$\Delta\varphi$	K_1 (km s^{-1})	γ_1 (km s^{-1})	K_2 (km s^{-1})	γ_2 (km s^{-1})
(all φ)	195 ± 7	21 ± 5	196 ± 7	0 ± 6
± 0.12	190 ± 6	24 ± 6	193 ± 7	1 ± 7
± 0.07	183 ± 5	24 ± 5	188 ± 7	1 ± 7
± 0.06	176 ± 4	25 ± 4	181 ± 7	1 ± 7

blended spectra. (Oddly enough, this peculiarity was misinterpreted by SDL as evidence for a considerably eccentric orbit, which they found inexplicable.)

Assuming circular orbits, we have recomputed the amplitudes K_1, K_2 by fitting sine curves to the radial velocities published by Mammano et al. (1974), taking into account different phase intervals around quadrature (Table 8). Proximity corrections (not included in Table 8) amount to $+4$ km s^{-1} in K_1, $+6$ km s^{-1} in K_2 near quadrature (these were evaluated using synthetic radial velocity curves computed with the WD program). We first note that consistently $K_2 > K_1$ as we should have if the spectroscopic primary is the more massive component (in fact, we obtain $q=1.03\pm0.03$). As expected, the amplitudes derived are very sensitive to the interval chosen and residuals show a suspicious trend with phase. Considering only the immediate regions of the velocity maxima (where the measurements are presumably most reliable), we derive an amplitude ratio $K_2/K_1=1.03\pm0.03$, corresponding to a spectroscopic mass ratio of $q=1.04$, if proximity corrections are taken into account. We can only roughly estimate the reduction in the radial velocity amplitudes caused by Balmer line blending. Judging from similar cases among early-type close binaries, it could amount 5–10%.

In the absence of a high-quality spectroscopic study, it is not possible to derive reliable absolute dimensions. So the values $K_1 \approx 190 \pm 15$ km s^{-1}, $q=1.05$, adopted for the following discussion, should be considered as a rough guess, at best. From these, we derive $M_1 \approx 5.0 M_\odot$ ($\pm 25\%$), $M_2 \approx 4.8 M_\odot$, and a mean $\log g$ of 3.86 ± 0.04 consistent with the value estimated from the Strömgren indices. Together with $\log \bar{M}=0.74\pm0.10$ and $\log \bar{T}_e=4.205\pm0.015$, this corresponds to a quite evolved stage on the main sequence. Further derived astrophysical parameters for the BF Aur system are given in Table 9.

Figure 7 shows a comparison with evolutionary tracks calculated on the standard model (Claret & Giménez 1989) and on models including convective core-overshooting (Maeder & Meynet 1988, 1989), for an adopted chemical composition of $(X, Z)=(0.70, 0.02)$. The steepness of the isochrones near the TAMS explains why components of somewhat different masses can have almost equal temperatures. The models by Maeder & Meynet include, as mentioned, convective core-overshooting (lengthening the main-sequence lifetime considerably) and seem to fit the properties of BF Aur better than the standard model. Note that the primary component should have evolved already beneath the TAMS if there were no overshooting.

Figure 3 shows the geometry of the binary BF Aur. The more massive component almost fills its Roche lobe. Whether BF Aur is truly semi-detached, is still an open question. At present there is no indication for any mass transfer or associated period change. From the $O-C$ diagram (Fig. 8), we conclude that the period has been essentially constant during the last 50 years. So it seems that the phase of rapid mass transfer has not yet started in BF Aur.

4. Conclusions

The application of the simplex algorithm to BF Aurigae shows that no a-priori assumptions on the geometries of close binaries are required in this case. However, application of the so-called

Table 9. Astrophysical data for BF Aur

	Primary	Secondary
P (day)	1.5832	
q	1.05 ± 0.05	
H (10^{52}g cm^{2}s^{-1})	16.3 ± 0.5	
h (10^{18} cm^{2}s^{-1})	33.3 ± 1.3	
lg J_c	-0.536 ± 0.02	
$M/M_\odot$	4.8 ± 0.5	5.0 ± 0.5
$R/R_\odot$	4.2 ± 0.2	4.7 ± 0.2
$\langle\rho\rangle$ (g cm^{-2})	0.091 ± 0.02	0.070 ± 0.02
lg g (cgs)	3.87 ± 0.03	3.80 ± 0.03
T_{eff} (K)	16000 ± 200	15970 ± 200
lg $L/L_\odot$	3.02 ± 0.1	3.11 ± 0.1
M_{bol}	-2.8 ± 0.1	-3.0 ± 0.1
$B.C.$	-1.5	-1.5
M_{vis}	-1.3 ± 0.1	-1.5 ± 0.1
$E(b-y)$	0.154 ± 0.01	
A_V	0.66 ± 0.01	
V_0	8.14 ± 0.01	
$(m-M)$	10.3 ± 0.15	
Distance (pc)	1150 ± 80	

[1] H = orbital angular momentum; h = angular momentum per unit of reduced mass; $J_c = q(1+q)^{-2}p^{1/3} \approx H/M^{5/3}$ specific angular momentum; $\langle\rho\rangle$ = mean stellar densities.

[2] We define as primary the component eclipsed at primary minimum (phase 0.0), which has, however, lower mass and luminosity, i.e. it is the spectroscopic secondary.

grid method (here in q) may be necessary in such systems to achieve a reliable solution. Our analysis of the BF Aur data, based on less restrictive assumptions than in SDL and avoiding the use of normal points, clarified the open question of the solution geometry (primary minimum is an occultation); a careful re-analysis of both the photometry and the spectroscopy gave $q=1.05 \pm 0.05$ and removed the former discrepancy between photometric and spectroscopic mass ratio.

BF Aur can now be understood as a pair of two evolved main-sequence stars of spectral type B5 V, of which the more massive component now almost fills its Roche lobe. This makes BF Aur useful as a test object for the importance of convective core-overshooting in main sequence evolution. Models including this effect seem to fit better the deduced properties of BF Aur. Although BF Aur may be on the verge of becoming an inverse Algol (the configuration suggested by SDL), interactions, while being clearly visible, are small scale and Roche lobe overflow probably has not yet fully developed.

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