International Journal of Production Research
Vol. 00, No. 00, 00 Month 2010, 1–17

RESEARCH ARTICLE

A hybrid general lot-sizing and scheduling formulation for a production process with a two-stage product structure

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(*received September 2009, revised April 2010, July 2010*)

Tailored for a complex application in the process industry, this paper examines a multi-product production planning and scheduling problem with sequence-dependent setup cost and times. The manufacturing process is characterized by a two-stage structure where the sequencing problem occurs on the first level and contribution margin, holding cost, penalty cost are accounted on the second level. We present a hybrid mixed-binary optimization model based on the General Lot-sizing and Scheduling Problem (GLSP) of Fleischmann and Meyr (1997), which combines discrete and continuous-time elements within a standard *Inventory and Lot-size* (I&L) formulation. Since the I&L formulation does not provide sharp LP-relaxation bounds, we present two alternative reformulations based on a transportation problem. In a numerical study inspired by real industry data we show that on average both reformulations yield significant improvements in computation time and integrality gap.

Keywords: Lot-sizing; scheduling; mixed-integer programming; reformulations; process industry

1. Introduction and literature review

Applications of quantitative models and computer-based planning systems have received considerable attention in the process industry. Since changeover operations are complex and expensive in time and cost, spreadsheet-based planning and scheduling quickly becomes inadequate and more sophisticated solutions are required. Advances in information technology and progress in the development of quantitative methods enabled many

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ISSN: 1478-6451 print/ISSN 1478-646X online
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DOI: 10.1080/1478645YYxxxxxxx
http://www.informaworld.com

successful implementations of advanced planning systems (APS) tailored to industry requirements and to support decision-making on strategic, tactical, and operational levels (Günther and Van Beek 2003). However, complex product and manufacturing structures are often not properly incorporated within standard APS so that customized planning models are required.

Simultaneous lot-sizing and sequencing problems with sequence-dependent setup cost and setup times have received a lot of attention in the literature. Drexl and Kimms (1997) summarize contributions up to 1996. A comprehensive overview is provided in Sürie (2005) and Jans and Degraeve (2008). An extensive overview of planning and scheduling problems with a focus on the process industry is given by Kallrath (2002).

The majority of lot-sizing and scheduling problems are formulated on a discrete time scale using either a *large-bucket* time scale where multiple items can be produced in each period, or a *small-bucket* time scale where only one item is produced in each period. The *Capacitated Lot Sizing and Scheduling Problem* (CLSP) is the most basic large-bucket lot-sizing problem where scheduling decisions are not incorporated are usually solved separately from the lot-sizing problem (Karimi et al. 2003). In order to integrate the lot-sizing and scheduling problem into large-bucket problems, Lasserre (1992) develops a two-stage integrated production planning and job-shop scheduling approach. Clark and Clark (2000) develop a mixed-integer programming formulation for a multi-product lot-sizing problem with sequence-dependent setup times where multiple setups per planning period are allowed. They test static and rolling-horizon formulations. Clark and Clark (2010) propose a lot-sizing and scheduling approach with sequence-dependent setup times at an animal nutrition plant, which is based on the *asymmetric traveling salesman problem* (ATSP). The *Discrete Lot Sizing and Scheduling Problem* (DLSP), introduced by Lasdon and Terjung (1971) is the fundamental small-bucket model. It provides the ability to completely integrate lot-sizing and scheduling decisions. Several papers have examined theoretical and computational aspects of the DLSP in general and of the DLSP with sequence dependencies (DLSDSD), among others Salomon et al. (1997), Fleischmann (1994), and Jordan and Drexl (1998). The primary limitation of using discrete time scales is the unnecessary increase of the overall problem size due to the introduction of additional binary variables associated with each discrete time interval. This inherent limitation has attracted the development of continuous time scale models. The basic idea is that beginning and/or ending of a period, usually defined as events, are endogenous variables with the consequence that the duration of periods is not necessarily equal. Due to the variability of events, the scheduling process becomes challenging since the mathematical model is more complicated compared to a discrete-time model. However, continuous-time approaches require much less computational effort compared to discrete-time models (Floudas and Lin 2004).

Hybrid models refer to a class of models which combine large-bucket and small-bucket time scales. The most general hybrid model is the *General Lot Sizing and Scheduling Problem* (GLSP) introduced by Fleischmann and Meyr (1997). The GLSP is essentially a large bucket model but it also includes an internal variable time scale within each particular large bucket period that determines the size and position of production lots. Koçlar and Süral (2005) show that the GLSP of Fleischmann and Meyr (1997) has the limitation in a way that the production state between two consecutive periods is only conserved if the available capacity exceeds the minimum production quantity. They generalize the model by a additional constraints. The most comprehensive study of the GLSP with sequence-dependent changeovers is provided by Koçlar (2005) who discusses the impact and validity of some commonly encountered assumptions.

It is commonly known that the conventional I&L modeling approach that uses production and inventory variables for every production period does not provide strong lower bounds in the LP relaxation for traditional capacitated lot-sizing and scheduling problems. Pochet and Wolsey (2006) provide the fundamental theory on reformulations for lot-sizing and scheduling problems. Models with variable redefinitions as a network representation are originally introduced by Krarup and Bilde (1977), Rosling (1986), and Eppen and Martin (1987). Many authors use their findings to investigate tighter formulations of the CLSP, e.g., Stadtler (1996) and Denizel and Süral (2006) ("tighter" in this context means that the respective LP relaxation is closer to the optimal objective value).

To the best of our knowledge Koçlar (2005) is the only study that analyzes reformulations of lot-sizing and planning problems in a GLSP context considering sequence-dependent setup costs and setup times. Koçlar (2005) presents an alternative formulation of the GLSP based on the Transportation Problem (TP) and tests LP-relaxation and MIP performance, and shows that the MIP performance is highly sensitive with respect to the parameter settings even for small test instances. However, the numerical experiments is confined to small test instances. Koçlar (2005) also investigates the impact of minimum production quantities. The results turn out that minimum production quantities do not affect the MIP performance for small test instances. However, we will show later that this result does not hold for real world test instances. Although a wide range of papers does exist that study lot-sizing and planning problems as well as the superiority of efficient reformulations, only few studies evaluate models and reformulations based on real industry data, e.g., Tempelmeier and Buschkühl (2008), Kimms and Motta Toledo (2003), and Burman and Gershwin (1996).

This paper is the result of a project with a chemical company, specifically, a surfactant manufacturer. The planning model contains a lot-sizing kernel where two subsequent product batches require sequence-dependent setup costs and setup times. We present a hybrid mixed-binary optimization model based on the General Lot-sizing and Scheduling Problem (GLSP). We propose two alternative reformulations derived from the *Transportation problem* (TP) formulation of Denizel and Süral (2006) and Koçlar (2005). The first reformulation, called *Quantity-based transportation problem* (QTP), disaggregates production variables of each period into separate variables related to the size of each period demand satisfied from the period's production (see e.g. Denizel and Süral (2006)). The *Proportional transportation problem* (PTP) disaggregates production variables in a similar way, however, instead of defining demand quantities, the PTP uses the proportion of a period's demand satisfied from the production in previous periods (see, e.g., Tempelmeier and Buschkühl (2008) or Sürie and Stadtler (2003)). A computational experiment using real industry data highlights the average superiority of the TP reformulations compared to the conventional I&L formulation, however, it also indicates that the TP reformulations do not always outperform the I&L formulation. Under tight runtime restrictions, the average superiority is not statistically significant.

The remainder of the paper is organized as follows. In Section 2 we describe the tailored *hybrid GLSP* model based on a conventional I&L formulation. Section 3 presents the two reformulations. Section 4 illustrates the complexity of the hybrid-GLSP formulation and shows results of a computational experiment using real industry data where the performance the hybrid GLSP and both TP reformulations is analyzed under various runtime restrictions. We also describe how the model has been integrated into the company's planning process. We summarize our work in Section 5.

2. Model

2.1. *Model assumptions and notation*

We formulate the lot-sizing and scheduling problem as a mixed-binary model and include several company-specific requirements. The production process is characterized by a two-stage product structure. On the first stage (product level), lot-sizing and sequencing for multiple products is planned. The production is characterized by sequence-dependent setup cost and setup times, and for each product a minimum lot-size has to be satisfied. During production, products are split up into one or multiple articles (article level). Articles differ in their respective contribution margin, capacity consumption, and holding cost.

We assume that K products are processed on a single resource with limited production capacity. The production process splits these products into I articles where each article is assigned to exactly one product. Products and articles are indexed by $k \in \mathscr{K} := \{1, \ldots, K\}$ and $i \in \mathscr{I} := \{1, \ldots, I\}$, respectively, with $K \leq I$. The product-article allocation is represented by an incidence matrix A_{ik} with $A_{ik} = 1$, if article i is an outcome of product k and $A_{ik} = 0$, otherwise. The finite planning horizon is divided into a two-level time scale represented by macro and micro periods. Macro periods, representing months, are denoted by $t \in \mathscr{T} := \{1, \ldots, T\}$ and are assumed to be fixed and equidistant. In any macro period t, multiple setup operations are allowed. However, the available production time, in the following defined as available capacity, is limited by C_t. D_{ti} denotes demand of article i, which is supposed to be available at the end of macro period t. In contrast to the macro period time scale, the micro period time scale, indexed by $s \in \mathscr{S} := \{1, \ldots, S\}$, is a continuous event-based representation of time and is characterized by variable and non-equidistant time periods. Any micro period s is defined by the time points τ_s and τ_{s-1}, the ending time of micro period s and $s-1$, respectively. Since the production capacity in macro period t is the available production time, the production capacity of a micro period s within a macro period t is the available production time within micro period s, i.e., $C_t(\tau_s - \tau_{s-1})$. In any micro period s only a single setup operation is allowed inducing that only one product can be produced.

At the beginning of each micro period s the machine is set up for a particular product k. Within each period s, first the production process of product k is completed before the changeover operation, if any, is performed. The length of s depends on the production time of product k as well as the sequence-dependent setup time if the machine is set up from product k to product l. P_k describes the capacity consumption to produce one unit of product k, Z_{kl} is the sequence-dependent setup time, and the sum of production and setup time must not exceed the production capacity of s. If a changeover operation to product k was performed in micro period $s-1$, the minimum production requirement of product k in period s is Q_k. If no setup operation is performed, the machine setup is carried over into the next micro period. We denote P_i as the capacity consumption to produce one unit of article i with $P_i = P_k$ for $A_{ik} = 1$.

As changeover times alone would not cover the full picture of reality related to changeovers (risk of startup, quality loss in part of the production, etc.), the number of changeover operations should be limited in each period to at most W setups. Accordingly, we assume that every period t contains exactly W micro periods and the total number of micro periods over the entire planning horizon is $S = WT$. The special case $W = K$ allows that all products can be produced within a macro period. This most general formulation provides the highest flexibility in terms of determining the optimal number of setups. Let the subset $\mathscr{S}_t \subset \mathscr{S}$ define the set of all micro periods s that form macro period t with $|\mathscr{S}_t| = W$. Furthermore, we define $\bar{\mathscr{S}} \subset \mathscr{S}$ with $\bar{\mathscr{S}} = \{W, 2W, \ldots, TW\}$ all micro periods s representing the last micro period of a macro period. To couple exogenous macro and endogenous micro period time scale, it is assumed that for all $s \in \bar{\mathscr{S}}$ the ending time τ_s is fixed and equal to the ending time of its macro period, i.e., $\tau_W = 1$, $\tau_{2W} = 2$.

Several company-specific requirements have been incorporated. According to the company's production philosophy, the quantity which satisfies the demand in a certain period has to be available at the end of the previous period. The underlying strategy having all demand available at the beginning of a month is to provide a higher flexibility and product availability. An implication of this philosophy is that the on-hand inventory at the beginning of the first production period contains the demand of this period, which is not element of the planning horizon. To determine the "actual" initial inventory level being available to satisfy demand of the planning horizon, the demand of the first production period has to be deducted from the initial on-hand inventory, which allows the initial inventory to be negative. Let $\bar{y}_{0i}$ define the initial inventory level of article i after deducting the first-period demand. For the sake of model feasibility, the model allows unsatisfied demand (unscheduled products). However, in order to penalize unsatisfied demand the company required to include penalty cost associated to each unit unsatisfied demand. Furthermore, the planning tool has to be able to fix production quantities in certain macro period. In these "fixed macro periods" only a predetermined and exogenously fixed production plan can be produced. Macro periods with fixed production schedules are denoted by $\mathscr{T}_f := \{1, \ldots, T_f\} \subseteq \mathscr{T}$, the set of "fixed macro periods". For any period $t_f \in \mathscr{T}_f$ production quantities of all products are exogenously given by $\bar{Q}_{t_f k} \geq 0$. Another requirement arose by a limited product availability. For example, if a certain product is known to be limited with the consequence that only 80% of the capacity can be utilized, the available production capacity for this particular product can be exogenously reduced to 80%. Formally, let $G_{tk} \in [0, 1]$ be the known unused fraction of capacity C_t for product k in macro period t. Hence, $(1 - G_{tk})C_t$ is the maximum capacity available to produce product k. If $G_{tk} = 0$, then the entire capacity is available for product k in period t, and if $G_{tk} = 1$ no capacity is available. We furthermore define $x(t)$ as the $t-$th entry of set x, e.g., $\bar{\mathscr{S}}(t)$ is the $t-$th entry of set $\bar{\mathscr{S}}$. We define the following revenue and cost parameters: R_i denotes the marginal revenue per unit of article i, V_{kl} denotes the sequence-dependent setup cost, if a setup operation from product k to product l is performed, H_i denotes the inventory holding cost per unit of article i occurring at the end of a macro period, and F_i denotes the penalty cost per unit lost sale of article i.

2.2. *Model*

We define the following decision variables:

q_{sk} : Production quantity of product k in micro period s,
p_{si} : Production quantity of article i in micro period s,
y_{ti} : Inventory level of article i at the end of macro period t,
x_{ti} : Sales of article i in macro period t,
γ_{sk} : Setup state variable; $\gamma_{sk} = 1$, if the system is set up for product k in micro period s; otherwise $\gamma_{sk} = 0$,
ξ_{skl} : Sequence-dependent setup variable; $\xi_{skl} = 1$, if a setup operation from product k to product l is performed in micro period s; otherwise $\xi_{skl} = 0$,
τ_s : Ending time of micro period s,
f_{0i} : Unsatisfied demand given a negative initial inventory, which could not be satisfied

in the first production period.

The objective is to maximize the total profit Π over the entire planning horizon and is expressed by (1).

$$\Pi = \underbrace{\sum_{t\in\mathscr{T}}\sum_{i\in\mathscr{I}} R_i x_{ti}}_{\text{Revenue}} - \underbrace{\sum_{s\in\mathscr{S}}\sum_{k\in\mathscr{K}}\sum_{l\in\mathscr{K}} V_{kl}\xi_{skl}}_{\text{Setup cost}} - \underbrace{\sum_{t\in\mathscr{T}}\sum_{i\in\mathscr{I}} H_i(y_{ti}+x_{ti})}_{\text{Holding cost}} \\ - \underbrace{\sum_{t\in\mathscr{T}}\sum_{i\in\mathscr{I}} F_i(D_{ti}-x_{ti}) - \sum_{i\in\mathscr{I}} F_i f_{0i}}_{\text{Penalty cost}} \quad (1)$$

Π is the total revenue minus total cost, i.e., inventory holding cost, setup cost, and penalty cost. Sequence-dependent setup cost are charged in every micro period s where a product changeover occurs. Holding cost are computed at the end of a macro period t. Unlike classic lot-sizing problems where the demand of a period is not included in the final inventory of this period, in our problem the demand, which has to be available at the end of period t is actually needed to satisfy demand of period $t+1$. Accordingly, holding cost are also charged to sales of period t. The first term of penalty costs represents cost that occur if the inventory level at the end of a macro period t does not cover demand. The second term represents penalty cost for unsatisfied demand out of a negative initial inventory which could not be satisfied in the first production period. The following constraints have to be taken into account.

Inventory balance equation

$$\bar{y}_{0i} + \sum_{s\in\mathscr{S}_1} p_{si} - x_{1i} + f_{0i} = y_{1i} \qquad \forall i \in \mathscr{I} \quad (2)$$

$$\bar{y}_{0i} + \sum_{s\in\mathscr{S}_1} p_{si} + f_{0i} \geq 0 \qquad \forall i \in \mathscr{I} \quad (3)$$

$$y_{t-1,i} + \sum_{s\in\mathscr{S}_t} p_{si} - x_{ti} = y_{ti} \qquad \forall t \in \mathscr{T}\backslash\{1\} \text{ and } i \in \mathscr{I} \quad (4)$$

Constraints (2) and (4) are standard inventory balance equations. Constraint (3) is necessary since we allow $\bar{y}_{0i}$ to be negative, i.e., there might be some unsatisfied demand from past periods, which can be produced in the first macro period. If there is no remaining capacity, this demand is lost and f_{0i} is positive.

Sales restriction

$$x_{ti} \leq D_{ti} \qquad \forall t \in \mathscr{T} \text{ and } i \in \mathscr{I} \quad (5)$$

The sales constraint ensures that sales of article i in macro period t cannot be larger than the demand of that particular period.

Capacity constraint of micro period s

$$\sum_{k\in\mathscr{K}} P_k q_{sk} + \sum_{k\in\mathscr{K}} \sum_{l\in\mathscr{K}} Z_{kl}\xi_{skl} \le C_t(\tau_s - \tau_{s-1}) \quad \forall t \in \mathscr{T} \text{ and } s \in \mathscr{S}_t \tag{6}$$

The available production capacity of every s is given by $C_t(\tau_s - \tau_{s-1})$, which is the fraction of the available capacity in macro period t used in micro period s. Constraint (6) ensures that the production time plus setup time never exceeds available capacity.

Coupling of endogenous micro-period and exogenous macro-period time scale

$$\tau_{\bar{\mathscr{S}}(t)} = t \quad \forall t \in \mathscr{T} \tag{7}$$

To couple micro periods and macro periods the ending time of a micro period s which is also the last period of macro period t has to be equal to t, the fixed ending time of macro period t.

Logic condition

$$q_{sk} \le C_t\gamma_{sk} \quad \forall t \in \mathscr{T},\ s \in \mathscr{S}_t, \text{ and } k \in \mathscr{K}, \tag{8}$$

This condition ensures that product k can only be produced in micro period s, if the system is set up on product k at the beginning of this micro period.

Minimum production requirement

$$q_{sk} \ge Q_k(\gamma_{sk} - \gamma_{s-1k}) \quad \forall s \in \mathscr{S}\backslash\bar{\mathscr{S}} \text{ and } k \in \mathscr{K}, \tag{9}$$

$$q_{sk} + q_{s+1k} \ge Q_k(\gamma_{sk} - \gamma_{s-1k}) \quad \forall s \in \bar{\mathscr{S}} \text{ and } k \in \mathscr{K}, \tag{10}$$

Constraints (9) and (10) ensure that after a product changeover from any arbitrary product to product k a lot of at least the minimum production quantity of Q_k units is produced. The original GLSP formulation of Fleischmann and Meyr (1997), which only contains constraint (9), is unable to find feasible solutions for cases when producing the minimum production quantity spills over two macro periods. In their formulation it is necessary to complete the minimum production quantity within a micro period, which is satisfied by constraint (9). Constraint (10) is necessary to ensure a continuous production process across macro periods, i.e., if the time interval of the last micro period within a macro period is not sufficient to produce the minimum quantity and production is continued in the next macro period, then the sum of two consecutive production quantities q_{sk} and q_{s+1k} has to satisfy the minimum production quantity. However, constraints (10) are only necessary for all $s \in \bar{\mathscr{S}}$ since these periods have a fixed ending time (Koçlar and Süral 2005).

Changeover logic

$$\gamma_{s-1k} + \gamma_{sl} \le \xi_{s-1kl} + 1 \quad \forall s \in \mathscr{S} \text{ and } k, l \in \mathscr{K} \tag{11}$$

This constraint satisfies that a changeover operation from product k to l is performed if the setup state in two consecutive micro periods is on k and l.

Product-article balance

$$q_{sk} = \sum_{i \in I} A_{ik} p_{si} \quad \forall s \in \mathscr{S} \text{ and } k \in \mathscr{K} \tag{12}$$

Constraints (12) ensure that the production quantity of product k in micro period s is equal to the sum of all associated articles produced in s.

Setup existence

$$\sum_{k \in \mathscr{K}} \gamma_{sk} = 1 \qquad \forall s \in \mathscr{S} \tag{13}$$

These constraints impose that a certain setup state must exist in every micro period s.

Limited production capacity available for each product

$$\sum_{s \in \mathscr{S}_t} P_k q_{sk} \leq (1 - G_{tk}) C_t \qquad \forall t \in \mathscr{T} \text{ and } k \in \mathscr{K} \tag{14}$$

These constraints exogenously limit the maximum available production capacity for each product in each period. If $G_{tk} = 0$, there is no additional capacity limitation and q_{sk} is bounded by (6) and (8).

Macro periods with fixed production quantities

$$\sum_{s \in \mathscr{S}_{t_f}} q_{sk} = \bar{Q}_{t_f k} \qquad \forall t_f \in \mathscr{T}_f \text{ and } k \in \mathscr{K} \tag{15}$$

Constraints (15) ensure that if a macro period t is element of $\mathscr{T}_f$, for any product k only exogenously given production quantities can be produced.

Binary and non-negativity constraints

$$\gamma_{sk} \in \{0, 1\} \qquad \forall s \in \mathscr{S} \text{ and } k \in \mathscr{K} \tag{16}$$

$$q_{sk},\ p_{si},\ x_{ti},\ y_{ti},\ \tau_s,\ \xi_{skl} \geq 0,\ f_{0i} \geq 0 \quad \forall t \in \mathscr{T},\ s \in \mathscr{S},\ k, l \in \mathscr{K},\ \text{and } i \in \mathscr{I} \tag{17}$$

The last two constraints, (16) and (17), define the domains of the binary and continuous variables, respectively. Although the variables ξ_{skl} have to be binary, it is sufficient to declare these variables as non-negative because objective function (1) and constraints (11) ensure the binary property.

3. Reformulation

In this section we consider two alternative reformulations of the hybrid GLSP formulated in Section 2. We use the concept of the *Transportation problem* (TP) reformulation of Denizel and Süral (2006) and Koçlar (2005). While the first reformulation (*quantity-based TP*) is a quantity-based disaggregation where production quantities are allocated to the required demand period (Denizel and Süral 2006), the second approach (*proportional-based TP*) redefines production variables as a proportion of demand that is produced in a particular period (Tempelmeier and Buschkühl 2008).

Quantity-based transportation problem (QTP)

The QTP disaggregates decision variables for every production quantity in micro period s related to the demand period when the quantity is required. This reformulation requires a transformation of production decisions on the article level. Let q_{sti} denote the amount of the article-i demand in macro period t, which is produced in micro period s, $s = 1, \dots, S$, $t = 1, \dots, T$, and $s \leq tW$ to exclude backordering. Therefore, production variables on product and article level are

$$p_{si} = \sum_{t=\lceil \frac{s}{W} \rceil}^{T} q_{sti} \quad \text{and} \quad q_{sk} = \sum_{i \in \mathscr{I}} \sum_{t=\lceil \frac{s}{W} \rceil}^{T} A_{ik} q_{sti}. \tag{18}$$

Moreover, we define $y0_{ti}$ as the demand of article i and macro period t which is satisfied from initial inventory. By this reformulation, sales and production variables can be replaced. Sales of article i in macro period t is the sum of the initial inventory reserved to satisfy demand in period t and the production quantities from micro period 1 to tW related to macro period t, i.e.,

$$x_{ti} = y0_{ti} + \sum_{s=1}^{tW} q_{sti}. \tag{19}$$

Moreover, there is no longer the need to have additional inventory decision variables since inventory can be replaced as follows:

$$y_{ti} = \bar{y}_{0i} - \sum_{l=1}^{(t-1)W} y0_{li} + \sum_{s=1}^{tW} \sum_{l=t}^{T} q_{sli}, \tag{20}$$

where $\left(\bar{y}_{0i} - \sum_{l=1}^{(t-1)W} y0_{li}\right)$ represents the initial inventory of article i after satisfying demand from period 1 to $t-1$. Thus, the optimization problem is:

$$\Pi_{QTP} = \sum_{t \in \mathscr{T}} \sum_{i \in \mathscr{I}} R_i \left(y0_{ti} + \sum_{s=1}^{tW} q_{sti} \right) - \sum_{t \in \mathscr{T}} \sum_{i \in \mathscr{I}} H_i \left(\bar{y}_{0i} - \sum_{l=1}^{(t-1)W} y0_{li} + \sum_{s=1}^{tW} \sum_{l=t}^{T} q_{sli} \right)$$
$$- \sum_{s \in \mathscr{S}} \sum_{k \in \mathscr{K}} \sum_{l \in \mathscr{K}} V_{kl} \xi_{skl} - \sum_{t \in \mathscr{T}} \sum_{i \in \mathscr{I}} F_i \left(D_{ti} - \left(y0_{ti} + \sum_{s=1}^{tW} q_{sti} \right) \right) - \sum_i F_i f_{0i} \tag{21}$$

$$s.t.\ \ y0_{1i} + \sum_{s\in\mathscr{S}_1} q_{s1i} + f_{0i} - D_{ti} = 0 \quad \forall i \in \mathscr{I} \tag{22}$$

$$\bar{y}_{0i} + \sum_{s\in\mathscr{S}_1} q_{s1i} + f_{0i} \geq 0 \ \forall i \in \mathscr{I} \tag{23}$$

$$\sum_{t\in\mathscr{T}} y0_{ti} \leq \bar{y}_{0i} \qquad \forall i \in \mathscr{I} \tag{24}$$

$$y0_{ti} + \sum_{s=1}^{tW} q_{sti} \leq D_{ti} \qquad \forall t \in \mathscr{T} \text{ and } i \in \mathscr{I} \tag{25}$$

$$\sum_{i\in\mathscr{I}} \sum_{l=\lceil \frac{s}{W} \rceil}^{T} P_i q_{sli} + \sum_{k\in\mathscr{K}} \sum_{l\in\mathscr{K}} Z_{kl}\xi_{skl} \leq C_t(\tau_s - \tau_{s-1}) \ \ \forall t \in \mathscr{T} \text{ and } \ s \in \mathscr{S}_t \tag{26}$$

$$\tau_{\mathscr{S}(t)} = t \qquad \forall t \in \mathscr{T} \tag{27}$$

$$A_{ik}q_{sti} \leq \gamma_{sk}C_t \qquad \forall s \in \mathscr{S},\ k \in \mathscr{K}, i \in \mathscr{I},\ \text{and } \ t \in \mathscr{T} \tag{28}$$

$$\sum_{i\in\mathscr{I}} \sum_{t=\lceil \frac{s}{W} \rceil}^{T} A_{ik}q_{sti} \geq Q_k(\gamma_{sk} - \gamma_{s-1k}) \qquad \forall s \in \mathscr{S}\backslash\bar{\mathscr{S}} \text{ and } k \in \mathscr{K} \tag{29}$$

$$\sum_{i\in\mathscr{I}} \sum_{t=\lceil \frac{s}{W} \rceil}^{T} A_{ik}(q_{sti} + q_{s+1ti}) \geq Q_k(\gamma_{sk} - \gamma_{s-1k}) \ \forall s \in \bar{\mathscr{S}} \text{ and } k \in \mathscr{K} \tag{30}$$

$$\gamma_{s-1k} + \gamma_{sl} \leq \xi_{s-1kl} + 1 \qquad \forall s \in \mathscr{S} \text{ and } k, l \in \mathscr{K} \tag{31}$$

$$\sum_{k\in\mathscr{K}} \gamma_{sk} = 1 \qquad \forall s \in \mathscr{S} \tag{32}$$

$$\sum_{s\in\mathscr{S}_t} P_i \sum_{i\in\mathscr{I}} \sum_{t=\lceil \frac{s}{W} \rceil}^{T} A_{ik}q_{sli} \leq (1 - G_{tk})C_t \qquad \forall t \in \mathscr{T} \text{ and } k \in \mathscr{K} \tag{33}$$

$$\sum_{s\in S_{t_f}} \sum_{i\in\mathscr{I}} \sum_{l=\lceil \frac{s}{W} \rceil}^{T} A_{ik}q_{sli} = \bar{Q}_{t_f k} \qquad \forall t_f \in \mathscr{T}_f \text{ and } k \in \mathscr{K} \tag{34}$$

$$\gamma_{sk} \in \{0, 1\} \qquad \forall s \in \mathscr{S} \text{ and } k \in \mathscr{K} \tag{35}$$

$$y0_{ti}, q_{sti}, \tau_s, \xi_{skl}, f_{0i} \geq 0 \quad \forall t \in \mathscr{T},\ s \in \mathscr{S},\ k, l \in \mathscr{K},\ \text{and } i \in \mathscr{I} \tag{36}$$

As in the previous section, the total profit (21) is the total revenue minus total costs. Constraints (22) and (23) are related to the inventory balance equations (2) and (3) but are reformulated. (22) ensures that for any article i the sales in macro period 1 plus lost sales is equal to demand. Moreover, constraint (23) ensures that there is no negative inventory at the end of macro period 1. Inventory balance equations (4) are not needed anymore since inventory variables are replaced by production variables (20). (24) makes sure that initial inventory sold over the planning horizon does not exceed the actual inventory level. Since f_{0i} and $y0_{ti}$ are non-negative for all i and t, by using (24) and the objective function (21) it can be seen that if $y0_{ti} > 0$ for at least one t, then $f_{0i} = 0$. Constraints (25) correspond to (5) with x_{ti} replaced by (19). (26) is the capacity constraint in a micro period s corresponding to (6) with q_{sk} replaced by (18).

There is no need for a reformulation of constraint (27). In constraint (28) q_{sk} is replaced by $A_{ik}q_{sti}$ with the effect that if article i is an outcome of product k, this article can only be produced in period s, if the system is set up on product k. Constraints (29)-(30) and (33)-(34) correspond to (9)-(10) and (12)-(13), respectively, with q_{sk} replaced by (18). Constraints (31), (32), (35), and (36) do not need to be reformulated.

Comparing the model dimensions of I&L and QTP formulation shows that the number of constraints is not affected by the reformulation. Both types of formulations comprise $2IT + 2S + T(K + 1) + 2SK + K^2S$ constraints (assuming we substitute q_{sk} by p_{si} in the I&L formulation as in (12) and there is no macro period with a fixed production schedule). The main distinguishing characteristics between I&L and QTP formulation is the number of decision variables, in particular, the number of production and inventory variables. The I&L formulation comprises IWT production variables on the article level and IT inventory variables. Hence, the total number of production and inventory variables is $IWT + IT$, which is linearly increasing with the number of setup operations W and the number of macro periods T. The QTP comprises $\frac{IWT}{2}(T + 1)$ production variables, but no inventory variables. It can be seen that the number of variables quadratically increases with the number of macro periods but it linearly increases with the number of allowable setup operations. Some algebraic manipulations allow a better comparison of the number of decision variables. The number of variables in the QTP formulation is $IWT + IT + \frac{IWT}{2}(T - 1 - \frac{2}{W})$ where the term $\frac{IWT}{2}(T - 1 - \frac{2}{W})$ is the additional number of decision variables used by the QTP formulation, which is strictly positive for $T > 2$ and $W \geq 2$. Hence, there is a trade-off between an increasing model complexity of the QTP because of an increasing number decision variables and a better performance subject to a stronger LP relaxation.

Proportional transportation problem (PTP)

The proportional transportation problem disaggregates production quantities related to the fraction of demand of that period in which the quantity is required. Let q'_{sti} denote the proportion of the demand of period t and product i produced in micro period s for $s = 1, \ldots, S$ and $t = 1, \ldots, T$. Compared to the QTP where decision variables define quantities, decision variables of the PTP define fractions of demand, which are real numbers in a range of [0,1]. We get the PTP by altering equations (21) - (36) of the QTP and substituting all decision variables q_{sti} by $q'_{sti}D_{ti}$ for all s, t, i. Although QTP and PTP are characterized by nearly equivalent formulations, the PTP provides a tighter bound because all q'_{sti} are bounded between 0 and 1.

4. Computational illustrations and implementation

We performed a computational experiment to compare the performance of I&L with QTP and PTP formulation. The experiment is based on confidential industry data which cannot be provided in detail. A typical optimization run is done for a planning horizon of 13 months, 15 products, and a product-article allocation which results in 24 articles. Information on demands, capacities, setup times, setup costs, profit margins, holding costs, unit production requirements, initial inventories, and minimum batch sizes are used exactly as given by the industry application. Sequence-dependent setup times of a single product changeover range between 0 and 5% of the monthly capacity. Production capacity required for producing the minimum batch size ranges between 3% and 14% of monthly capacity. Typically, no more than 4 product setups are allowed per month. Setup and penalty cost are set equal to zero. Furthermore, capacity reductions as well as periods with fixed production quantities are not considered here. The computational complexity of lot-sizing and planning problems with sequence-dependent setup cost and setup times is strongly sensitive to various parameters and their dependencies, e.g., monthly capacity, length of the planning horizon, number of allowable product setups, product structure (number of products, articles, and their allocation), demand characteristics (sparse or dense demand matrix), as well as sequence-dependent setup times and costs (Koçlar 2005). For the numerical design we identified eight main factors that influence the performance and selected three factor levels (low, medium, high) for each factor. Table 1 shows the selection of parameters derived from the base level instance. Running a full factorial parameter analysis with $3^8 = 6,561$ instances is computationally expensive, so that we are interested in selecting a representative sub-sample out of these $6,561$ configurations. While this is unlikely to be achieved by random selection, we adopt a Latin Hypercube Design (LHD) to find parameter configurations, which are as different from one another as possible.

LHDs are spacefilling designs where the minimum distance between all design points, i.e., factor combinations, is maximized (Chen et al. 2006). With this problem being NP hard, we resort to the heuristic proposed by Grosso et al. (2009) to obtain an LHD with 21 factor combinations (also available online at http://www.spacefillingdesigns.nl). Since this LHD comes with 21 different factor levels $0, ..., 20$, we divide each factor level by seven and round down to the next integer to obtain factor levels $\{0, 1, 2\} \rightarrow \{$low,medium,high$\}$. While the resulting LHD may not be optimal with respect to the maximin criterion (maximizing the minimum distance), design points are sufficiently different to obtain a representative sample. The resulting detailed experimental design of the 21 test instances is presented in Table 1. For testing the performance of the three different model formulations, we implemented each model in Xpress Mosel 2.4 and solved it with Xpress Optimizer 19.0 on an Intel Core2Duo with 2.33 GHz and 2 GB memory. Default options concerning presolving and branch and cut options were used as provided by the software and not altered. For a typical optimization run with 13 months, 15 products, 24 articles, and 4 allowed setups per month, the hybrid GLSP has 23,585 constraints and 15,162 variables, 780 of which are binary. To evaluate the reformulations we carried out computational experiments with different runtime restrictions of 600, 1800, and 5400 seconds after which the solution process of the MIP solver was terminated. As a further termination criterion, we stopped the model optimization as soon as the integrality gap dropped below 0.5% and returned the current best solution.

Table 2 shows detailed performance results obtained with the I&L, QTP, and PTP formulation over all 21 test instances and for the three cut-off times of 600, 1,800, and 5,400 seconds. For any of the three cut-off times we observed that the I&L formulation is outperformed by at least one TP reformulation. In contrast to the I&L formulation, the QTP and PTP approaches found solutions with an integrality gap smaller than 0.5% for several test instances. The QTP approach was able to find solutions with a gap below 0.5% in four (6,14,18,21) out of the 21 test instances within 60 seconds. In three out of these four instances the solution was even found within 10 seconds. An increase in runtime, however, did not yield significant performance improvements. For instance, an increase of the time limit from 600 to 1800 seconds decreases the mean gap by only 0.48%. A further increase to 5400 seconds decreases the mean gap by another 0.3% (see Table 3). All four test instances have in common that the number of setups and the number of products is low or medium. Both parameters directly influence the model

Run	Capacity level	Number setups	Minimum lot size	Profit margin	Demand	No. of products	Setup times	Holding cost
1	low	medium	high	medium	medium	medium	medium	high
2	low	high	low	medium	medium	high	high	high
3	low	low	medium	high	high	high	medium	low
4	low	high	medium	low	medium	medium	low	low
5	low	high	medium	medium	low	low	high	medium
6	low	low	low	medium	low	low	low	high
7	low	medium	medium	low	high	low	high	low
8	medium	low	high	low	low	medium	medium	low
9	medium	high	high	high	high	high	medium	medium
10	medium	high	medium	low	low	high	medium	medium
11	medium	low	medium	high	low	medium	high	high
12	medium	medium	high	high	medium	low	medium	low
13	medium	medium	low	low	high	high	low	high
14	medium	medium	low	high	low	medium	high	low
15	high	medium	medium	high	high	low	medium	high
16	high	low	high	medium	medium	high	low	medium
17	high	high	low	high	low	medium	low	medium
18	high	low	low	low	medium	medium	high	high
19	high	medium	high	medium	high	high	high	medium
20	high	high	high	low	medium	low	low	medium
21	high	low	low	medium	high	low	low	low

Table 1. LHD design of 21 test instances out of 3^8 factor combinations

size, i.e., the number of variables and constraints. While QTP could solve only four test instances with a gap below 0.5%, the PTP approach solved six instances within 600 seconds, seven instances within 1800 seconds, and eight instances within 5400 seconds. Even when the average PTP performance is better than the QTP performance, there are four test instances (8,9,12,16) where the PTP was outperformed by the QTP. All four test instances have in common that products require large minimum lot-sizes, i.e. for production processes with large minimum lot-sizes, the QTP formulation seems to be more appropriate.

Table 3 compares the average performance across all formulations. Column 3 shows the mean integrality gaps for cut-off times of 600, 1,800, and 5,400 seconds, column 4 shows the paired differences between the I&L formulation and the two reformulations, and column 5 shows the paired differences between the two reformulations. For the paired differences, we report standard deviation and t-values together with the results of the respective t-test. Paired differences are significant when $p < 0.1$ (*), $p < 0.05$ (**) and $p < 0.01$ (***) or not significant (n.s.). We see that if cut-off times are low (600 sec.), the PTP is outperformed by the QTP and the difference between I&L and PTP is not significant. Moreover, the PTP pays off for longer runtimes where it significantly outperforms the QTP, i.e. at 1800 seconds ($p < 0.1$) and 5400 seconds ($p < 0.01$). This effect can be explained by the larger number of Simplex iterations necessary for the PTP.

Run	**600 sec.**						**1800 sec.**						**5400 sec**					
	Hybrid-GLSP		QTP		PTP		Hybrid-GLSP		QTP		PTP		Hybrid-GLSP		QTP		PTP	
	GAP in %	Time in sec.	GAP in %	Time in sec.	GAP in %	Time in sec.	GAP in %	Time in sec.	GAP in %	Time in sec.	GAP in %	Time in sec.	GAP in %	Time in sec.	GAP in %	Time in sec.	GAP in %	Time in sec.
1	8.92	600	4.38	600	3.74	600	8.00	1800	3.38	1800	3.40	1800	7.96	5400	3.20	5400	3.06	5400
2	3.84	600	3.10	600	0.36	86.81	3.73	1800	2.57	1800	0.36	86.53	3.73	5400	2.49	5400	0.36	87.08
3	8.04	600	4.38	600	3.87	600	7.99	1800	4.06	1800	3.34	1800	7.40	5400	4.00	5400	3.09	5400
4	3.21	600	2.54	600	1.85	600	3.02	1800	2.40	1800	1.74	1800	2.65	5400	2.23	5400	1.65	5400
5	14.71	600	8.95	600	8.77	600	13.55	1800	8.61	1800	7.78	1800	13.47	5400	8.15	5400	7.55	5400
6	2.40	600	0.43	6.86	0.16	15.19	2.22	1800	0.43	6.88	0.16	15.11	2.18	5400	0.43	7.14	0.16	15.03
7	1.00	600	0.83	600	0.55	600	0.97	1800	0.78	1800	0.50	981.03	0.89	5400	0.76	5400	0.49	979.87
8	14.21	600	3.62	600	4.56	600	13.90	1800	3.22	1800	3.96	1800	13.54	5400	2.91	5400	3.16	5400
9	11.99	600	10.49	600	68.10	600	9.45	1800	8.44	1800	10.90	1800	9.19	5400	7.67	5400	7.34	5400
10	28.83	600	19.48	600	17.68	600	24.29	1800	17.51	1800	17.09	1800	23.99	5400	16.22	5400	15.35	5400
11	7.57	600	1.49	600	1.36	600	7.35	1800	1.33	1800	1.04	1800	6.94	5400	1.26	5400	1.00	5400
12	2.45	600	1.27	600	1.37	600	2.26	1800	1.08	1800	1.18	1800	2.00	5400	1.01	5400	1.07	5400
13	5.77	600	4.39	600	0.36	12.86	5.72	1800	3.97	1800	0.36	13.02	4.88	5400	3.89	5400	0.36	13.03
14	0.82	600	0.50	55.81	0.37	15.19	0.76	1800	0.50	56.39	0.37	15.58	0.69	5400	0.50	56.75	0.37	15.03
15	1.01	600	0.94	600	0.61	600	0.96	1800	0.87	1800	0.54	1800	0.95	5400	0.85	5400	0.49	5249
16	11.38	600	0.85	600	0.80	600	10.73	1800	0.61	1800	0.69	1800	10.19	5400	0.52	5400	0.65	5400
17	2.01	600	1.54	600	1.16	600	1.85	1800	1.34	1800	1.01	1800	1.85	5400	1.27	5400	0.97	5400
18	4.94	600	0.43	5.47	0.00	2.09	4.52	1800	0.43	5.55	0.00	2.11	4.43	5400	0.43	5.47	0.00	2.05
19	13.17	600	12.48	600	8.61	600	11.35	1800	11.39	1800	8.31	1800	10.83	5400	10.32	5400	8.28	5400
20	21.00	600	15.14	600	12.31	600	20.49	1800	14.30	1800	11.90	1800	18.67	5400	13.38	5400	11.17	5400
21	0.64	600	0.24	4.94	0.31	3.36	0.61	1800	0.24	5.14	0.31	3.42	0.59	5400	0.24	5.08	0.31	3.31

Table 2. Numerical results

Time (sec.)	Model #	Mean Gap μ (in %)	Paired Differences $\mu_{\#} - \mu_{GLSP}$	St. Dev.	t	$\mu_{\#} - \mu_{QTP}$	St. Dev.	t
600	I&L	8.00						
	QTP	4.64	−3.35	3.50	−4.40 $^{(***)}$			
	PTP	6.52	−1.48	13.64	−0.59 $^{(n.s.)}$	1.88	12.84	0.67 $^{(n.s.)}$
1800	I&L	7.32						
	QTP	4.16	−3.16	3.29	−4.39 $^{(***)}$			
	PTP	3.57	−3.75	3.36	−5.12 $^{(***)}$	−0.60	1.32	−2.07 $^{(*)}$
5400	I&L	7.00						
	QTP	3.86	−3.14	3.23	−4.45 $^{(***)}$			
	PTP	3.19	−3.82	3.21	−5.45 $^{(***)}$	−0.68	0.98	−3.16 $^{(***)}$

Table 3. Means and paired differences of the integrality gaps for the three model formulations. (t-test: significant difference when $p < 0.1$ (*), $p < 0.05$ (**), $p < 0.01$ (***) and $n.s.$ when the difference is not significant.)

The implemented planning model is used on a regular basis once a month to support the short to mid-term rough cut planning. Currently, the company is using the I&L formulation. However, an adjustment of the planning tool to the alternative reformulations would be easy to obtain. Before implementation, the planning and scheduling process was only supported by manual methods as spreadsheet modeling, whiteboard analysis, and the experience of production planners. The planner manually evaluated a finite number of production alternatives and designed the production schedule to his or her best knowledge and experience so that due to the problem complexity only a small number of alternatives could be evaluated. Manual planning activities also complicated the coordination between production and sales division. The implemented model allows the company to generate optimal plans and to keep track of resulting costs. Short computation times facilitate communication with marketing and sales managers when it comes to backlog management and production priorities. Considering problem complexity and computational tractability of lot-sizing and scheduling problems with sequence-dependent setup times including minimum batch sizes, an average integrality gap of under 5% (using PTP with at least 1,800 sec. cut-off time) provides a reasonable starting point for detailed short-term planning. The model enables the company to calculate the impact of price and sales promotions on production schedules. Finally, the company declared that the implemented model creates more stability in the planning process, e.g., the model can be used by less experienced managers to generate alternative plans.

To integrate the model into the planning process, we created an interface between Xpress and Microsoft Excel using Visual Basic. The supply chain planner can now use Excel for planning the production campaign without getting involved with the Xpress modelling language, which increased usability and acceptance.

5. Conclusion

We developed a tailored hybrid general lot-sizing and scheduling model based on the GLSP for a complex practical problem from the process industry. Derived from the conventional I&L formulation incorporating several company-specific features, computational performance was enhanced by two reformulations based on the Transportation Problem. The reported numerical study gave an illustration of the performance of the hybrid GLSP and the two reformulations with respect to computational time and integrality gap for a real world lot-sizing and scheduling problem. The major findings are that for short cut-off times, the reformulations did not perform significantly better than the conventional I&L formulation. However, for a longer runtime the performance of both reformulations improved significantly. The results also indicated that minimum lot-sizes are an important performance driver. The planning tool is used by the company on a regular basis once a month.

For future work, it is worthwhile to investigate whether the inclusion of valid inequalities will help to further improve the performance of the reformulations. Moreover, the applicability of the GLSP framework together with an exact mixed integer programming solver is rather limited, especially if the number of products and changeovers and thus the number of micro periods increases. As a natural next step, decomposition approaches need to be developed to extend this approach to larger and more detailed problems.

Acknowledgements

We gratefully acknowledge the helpful comments of two anonymous referees.

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