Energy Portfolio Optimization for Electric Utilities: Case Study for Germany

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Summary. We discuss a portfolio optimization problem occurring in the energy market. Energy distributing public services have to decide how much of the requested energy demand has to be produced in their own power plant, and which complementary amount has to be bought from the spot market and from load following contracts. This problem is formulated as a mixed-integer linear programming problem and implemented in GAMS. The formulation is applied to real data of a German electricity distributor.

Key words: unit commitment; economic dispatch; portfolio optimization; power plant control; day-ahead market, MILP, GAMS

1 Introduction

We consider large German public services distributing energy in the order of magnitude of Düsseldorf, Hanover or Munich. On the one hand, the public services have to be large enough in order to utilize the optimization techniques discussed here but on the other hand they have to be smaller than the supra-regional electric distributor, *i.e.* RWE or E.ON.

The major difference of public services to supra-regional electric distributors is that public services usually do not sell excess energy in the energy market. They are price takers and their objective is to minimize the cost while meeting the demand for energy or electric power, resp.; in this paper we treat energy (physical unit: Wh or MWh) and electric power (physical unit: W or MW) as two different utilities which can be traded in the market.

The optimization model discussed in this article also does not apply to small public utility companies as they usually have one exclusive supplier of vendor, *i.e.* RWE or E.ON. Therefore, they do not have a portfolio of sources of supply which can be optimized.

The considered electric distributor has several sources of supply in order to satisfy the demand for power of their customers. Among these possibilities are:

- The electric power generation in a single power plant operated autarkic by the electric distributor.
- The electric power generation in an external power plant. The operation of the plant is regulated by the carrier to a great extend.
- The purchase of energy in arbitrary quantities at any time from a business partner, known by name, with a bilateral treaty. This form of trading is called "Over The Counter." It stands in contrast to the anonymous stock jobbing.
- The purchase of standardized power products on the stock exchange, in the so-called *spot market*, abbreviated by SM. This is short term trading.
- The purchase of power on the stock exchange in the forward market. This is long term trading.
- The purchase of power in arbitrary quantities though so-called *load following contracts* or short LFCs.

The complete range of the opportunities can only be exploited in the long-run; for instance in an optimization over the whole year. In this article, we focus on the short-term portfolio optimization; *i.e.* within one or two days. That is, we are given the operating conditions including the long-run decisions. The task is then to optimize the power plant operation and the purchase of energy in such a way that the total cost are minimized while satisfying the demand. The energy demand is given via a power forecast for the following day.

In this article, we develop a mixed-integer linear programming (MILP) formulation for the energy portfolio optimization problem allowing the following three sources of energy supply:

- The electric power generation in the own power plant,
- the purchase of standardized products from the spot market, and
- the purchase of power via the load following contracts with one supplier of vendor.

The mathematical programming formulation is implemented in the modeling language GAMS. The code has been added to the GAMS model library with the name `poutil.gms` (Portfolio Optimization for electric UTILities) [16].

This electricity optimization problem falls in the scope of the *unit commitment problem* and *economic dispatch problem*. In contrast to the unit commitment problem, our model does not include any constraints on the power transmission, reverse spinning or ramping. The economic dispatch problem differs from ours in the way that the different energy sources are only subject to capacity constraints whereas we have to deal with additional technical or production restrictions such as minimum idle time periods of the plant.

Dillon et al. [12] provide a mixed-integer linear programming formulation of the unit commitment problem, also taking into account energy exchange contracts. The model by CARRION and ARROYO [9] for thermal plants uses less binary variables than the model by DILLON. Our model assumes a discrete cost structure for the power plant in contrast to the quadratic one discussed by CARRION and ARROYO. Mixed integer programming was also used by HOBBS et al. [20] to solve the unit commitment problem. The optimal selling of energy in the electricity spot market is modeled as an MILP problem by ARROYO and CONEJO [1] and as a stochastic program by PHILPOTT and SCHULTZ [30]. In the literature, there are many specialized algorithms for solving the unit commitment problem [43, 35, 29, 34, 3] and the economic dispatch problem [26, 10, 11].

As we do a day-ahead planning, we assume that all data are reasonably well known. The day-ahead forecast is rather accurate but nevertheless subject to uncertainties. The forecast is derived from historical data, annual load profiles, weekday specifics tendencies, temperature profiles for the next days, and considers public holidays as well as special events such a soccer finals, formula I racings etc. Smoothing and averaging over many influence factors leads to a rather stable forecast. The remaining uncertainties are of the order of a few percent and may lead to minor changes; they are mostly covered by load following contract costs. The prices for the purchased energy are given through contracts and the spot market. Furthermore, we assume that we have a quite accurate power forecast for the planning horizon. However, when such data are not reliable or when looking at longer planning horizons, a stochastic model would be preferable against a deterministic one; taking into account for instance the stochastic spot prices and/or stochastic demand. Such models and algorithms are discussed, for instance, in [38, 39, 36]. Including hydro, wind or solar as an energy source into the model leads also to stochastic components [27, 17, 5].

A simple unit commitment model code is available in the LINGO library model `unitcom1.lg4` [24, 23].

We start with the description of the problem in Section 2. The mathematical formulations are discussed in detail in Section 3 including the special cost structure of the different energy sources and the constraints associated with the power plant operation. In Section 4, we discuss some limitations of the model and provide possible modifications of the formulation. Computational results for the implemented model in GAMS are given in Section 5. Conclusions of this article are provided in Section 6.

Throughout the article, we will introduce several sets, variables and input data given. We denote all variables with small letters and input data as capital ones. In the Appendices, all sets (App. A), variables (App. B), constraints (App. C), input data and parameters (App. D) used in the mathematical model are summarized along with their synonym in the GAMS model `poutil.gms` which is included in the GAMS model library [16].

2 Description of the Problem

In this section, we discuss the short term optimization problem for the day-ahead planning of the energy portfolio.

In general, the power curve of one day is given by the continuous function

$$P(t) \quad , \qquad 0 \leq t \leq 24 \quad ,$$

given in MW. We brake the power process into quarters of an hour. The use of quarter-hour-values as general time frame is a common standard in worldwide energy economics; furthermore it is based on several directives, as, for instance, in Germany the MeteringCode [42], in Austria a statistical regulation [28]; as a practical example one can find the published maximum load values of Stadtwerke Saarlouis GmbH in quarter hours [37]. Furthermore, in the energy industry, the continuous process of the produced and provided power is treated as fixed within a quarter-hour basis. With this convention, we can approximate the power curve through a step function. Let $\mathcal{T}$ be the set of quarter-hour time slices per day; *i.e.*, $\mathcal{T} := \{1, \ldots, N^{\mathrm{T}} = 96\}$. We assume that we are given the forecast of electric power for all 96 quarter-hour time intervals per day

$$P_t \quad , \qquad t = 1, \ldots, N^{\mathrm{T}} \quad ,$$

measured in MW. In order to meet the demand, the utility company disposes of three sources of supply,

- a power plant (PP) with given capacity,
- the opportunity to buy power from the spot market at the energy bourse in form of standardized products, and
- a load following contract with one supplier of vendor. The amount of energy is assumed to be unlimited.

The total cost for the fulfillment of the demand is then given by the sum of the power plant operation cost, the cost for the purchase of power from the spot market and the cost for the purchase of power from the open supply contract.

The structure of the cost components and the constraints involved are discussed in the following sections.

2.1 Power Plant Usage

We assume that we are given a natural gas power plant. The reasons are that they are quite common in Germany (23% of primary energy supply in 2004 [15]) and that they can be operated very flexibly. This implies that we do not have to consider restrictions which last for more then one day.

The costs of the power generation in the own power plant consist in principle of the fix costs per day and the variable costs per MWh generated. To simplify matters, the variable costs of the power generation are assumed to be constant. This disregards that operational costs depend on the actual degree of efficiency and that operating a power plant beside the point of optimum causes increasing variable costs; see Section 4.2 for further details.

Let us now discuss the constraints associated with the power plant usage. The power plant has a maximal power of $P^{\text{PP}}_{\max}$, measured in MW. During normal operation, the power plant should not be operated with less than 40% of its maximal power. This is not a technical restriction or a generally accepted convention, but a useful approach to avoid an obvious contradiction to the assumption of constant variable costs.

Let p^{PP}_t be the amount of power in MW of the power plant at time period t. Then we get

$$p^{\text{PP}}_t \geq 0.4 P^{\text{PP}}_{\max} \quad , \qquad \forall t, \tag{1}$$

in case the power plant is used; otherwise we have $p^{\text{PP}}_t = 0$, obviously.

For technical reasons, the power of the plant is not a continuous variable but fixed in steps of 10% of the maximal power. A restriction to 10% steps while running a power-plant is obviously deliberate but shall remember that an operator would never choose an infinite continuum of steps but only a small number of usual operating points. These so-called *partial load operation points* are ordinarily determined by technical attributes of the power plant and are supposed to be given. Whether these in our model are defined as equidistant steps or as a set of given figures does not matter. However, it is important to define them as a small set of discrete numbers to approach reality.

Define stage 1 as the idle stage of the plant and stages 2, 3, ..., 8 as the stages corresponding to the power level of 40% $P^{\text{PP}}_{\max}$, 50% $P^{\text{PP}}_{\max}$, ..., 100% $P^{\text{PP}}_{\max}$. The stages and the corresponding power level with respect to the maximal power level $P^{\text{PP}}_{\max}$ are illustrated in Figure 1. This allows us to substitute (1) by

$$p^{\text{PP}}_t = 0.1(\alpha_s + 2) P^{\text{PP}}_{\max} \quad , \qquad \forall t \tag{2}$$

with $\alpha_s \in \{2, 3, 4, 5, 6, 7, 8\}$.

In order to avoid permanent changes of the power level, we require any power stage to continue for at least D^{PP}_{act} quarter hours, with a typical value of $D^{PP}_{\text{act}} = 8$. A constant operation over a period of $D^{\min}_{\text{act}}$ quarter hours is a deliberate simplification of the model as well; but it covers the experience that it could be considered as ineffective to change the operation mode of an engine permanently. The change itself causes loss of energy through *start up-* and *shut down-losses* [45] which we do not want to take into consideration here. This restriction on the changes of the power plant can be formulated as

$$p^{\text{PP}}_j = p^{\text{PP}}_{j+1} = \ldots = p^{\text{PP}}_{j+k} \quad , \quad \text{with } k \geq 7 \quad , \tag{3}$$

where j is a time interval containing a shift of the power level.

To avoid a complete shut down of the power plant for only a short time period, any idle period has to last for at least 4 hours:

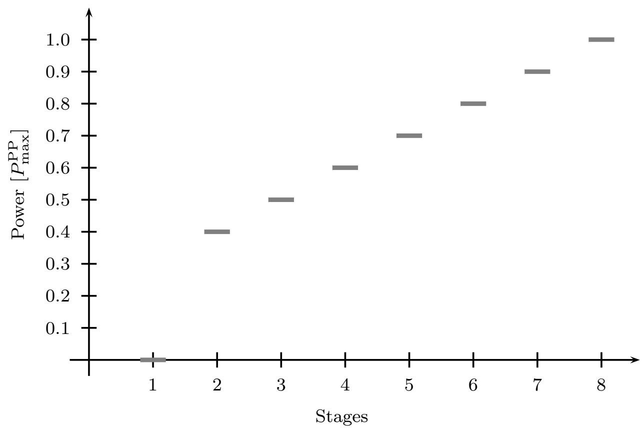

Fig. 1. Stages of the power plant vs. fraction of maximal power level

$$p_j^{\mathrm{PP}} = p_{j+1}^{\mathrm{PP}} = \ldots = p_{j+m}^{\mathrm{PP}} \quad , \quad \text{with } m \geq 15 \quad , \tag{4}$$

where j is a time interval containing an idle time.

We relax this condition for the end of a day. The idle times can then be shorter, as some part of this time can be transformed to the next day or coming from the previous one. These boundary conditions show the drawback of looking at each day separately. In reality, every day has some pre-history, providing the boundary conditions.

2.2 Energy Purchase from the Spot Market

The European Energy Exchange (EEX) in Leipzig provides the spot market as an opportunity to trade energy. This means that we can buy standardized products in short-term. We consider here the so-called *base load* and *peak load* contracts which belong to the continuous trading of EEX[3] [25]. They are traded at one day and delivered at the next day [13]. Special cases occurring for instance on weekends are not considered here; those are the weekend-base load contracts[4].

Each base load contract specifies the delivery of a constant power of 1 MW from 0:00am to 12:00pm at the following day after the completion of the contract.

Each peak load contract specifies the delivery of a constant power of 1 MW from 8:00am to 8:00pm at the following day after the completion of the contract.

[3] We do not consider selling in the auction market in our model.

[4] Weekend-base load contracts specify the delivery for 48h, starting at Saturday 0:00am and ending on Sunday 12:00pm; peak load-contracts for the weekends are not offered

Provider and customer remain anonymous for this contracts. The commercial clearing and settlement is handled by the EEX while the technical delivery is done through the power grid operators in Germany. Currently, the power grid in Germany is not uniform nationwide. There are four transmission network operators: E.ON, Vattenfall, RWE Transportnetz Strom and EnBW [41].

We get from the conventions above that the contribution to the energy portfolio from the spot market, e_t^{SM}, is given though the number α of base load and the number β of peak load contracts bought, while respecting the above time intervals for energy delivered.

The cost for the energy from the spot market is calculated via the total delivered energy amount in MWh.

2.3 Energy Purchase from the Load Following Contract

The load following contract can be seen as a compensation for the vacancy of the previously discussed sources of energy supply [19]. An energy load can be covered only partially by the standardized products from the spot market and the relatively inflexible power plant operation. However, the utility company is committed to meet the power demand of its customers. Therefore, the vacancy has to be closed by a flexible instrument. Obviously, the flexibility of this instrument makes the energy purchase from the load following contract to the most expensive source of the three discussed in the paper as it transfers all risk from the customer to the seller of the contract. The load following contracts are also called *full requirements contracts*.

The costs for the load following contract are determined via the typical two-component supply-contracts [14]. That is, the delivered power, or more precise the power level peak, as well as the delivered energy amount, are considered. In other words, it is the sum of the so-called *power rate* [€/MW] and the *energy rate* [€/MWh].

The power rate $C_{\mathrm{PR}}^{\mathrm{LFC}}$ of the load following contract is based on the highest drain of power (quarter-hour value) within a year $p_{\max}^{\mathrm{LFC}}$. To avoid random anomalies up to a certain amount, one usually applies the arithmetic mean of the two – in some contracts also three – highest monthly peaks as the rated value of the calculation of the power rate. We get for the cost of the power rate

$$C_{\mathrm{PR}}^{\mathrm{LFC}} = C_{\mathrm{PR,year}}^{\mathrm{LFC}} \cdot p_{\max}^{\mathrm{LFC}} \quad , \tag{5}$$

where $C_{\mathrm{PR,year}}^{\mathrm{LFC}}$ are the cost coefficient per MW of the power rate on an annual basis.

For the demand rate contracts considered in this article, usually there are defined annually quantity zones with different prices. Let Z_1 and Z_2 be the borders of the quantity zones given in MWh and let P_1^{LFC}, P_2^{LFC} and P_3^{LFC} be the prices in € per MWh in these zones. We denote by $e_{\mathrm{year}}^{\mathrm{LFC}}$ the delivered energy amount annually. Then, the prices in € per MWh are given by

$$\left\{\begin{array}{l} P_1^{\mathrm{LFC}},\ \text{if } 0 \leq e_{\mathrm{year}}^{\mathrm{LFC}} \leq Z_1 \\ P_2^{\mathrm{LFC}},\ \text{if } Z_1 < e_{\mathrm{year}}^{\mathrm{LFC}} \leq Z_2 \\ P_3^{\mathrm{LFC}},\ \text{if } Z_2 < e_{\mathrm{year}}^{\mathrm{LFC}} \end{array}\right\} \quad .$$

Recognize that the price P_1^{LFC} is paid for the amount of energy in zone 1, where price P_2^{LFC} is only paid for the amount of energy within zone 2, exceeding the quantity zone 1.

The quantity price $P_{\mathrm{year}}^{\mathrm{LFC}}$, or total variable cost per year associated with the LFC, can then be stated as

$$P_{\mathrm{year}}^{\mathrm{LFC}} = \begin{cases} P_1^{\mathrm{LFC}} \cdot e_{\mathrm{year}}^{\mathrm{LFC}}, & \text{if } 0 \leq e_{\mathrm{year}}^{\mathrm{LFC}} \leq Z_1 \\ P_1^{\mathrm{LFC}} \cdot Z_1 + P_2^{\mathrm{LFC}}\left(e_{\mathrm{year}}^{\mathrm{LFC}} - Z_1\right), & \text{if } Z_1 < e_{\mathrm{year}}^{\mathrm{LFC}} \leq Z_2 \\ P_1^{\mathrm{LFC}} \cdot Z_1 + P_2^{\mathrm{LFC}}\left(Z_2 - Z_1\right) + P_3^{\mathrm{LFC}}\left(e_{\mathrm{year}}^{\mathrm{LFC}} - Z_2\right), & \text{if } Z_2 < e_{\mathrm{year}}^{\mathrm{LFC}} \end{cases} \quad .$$

The resulting piece-wise linear price curve is shown in Fig. 2.

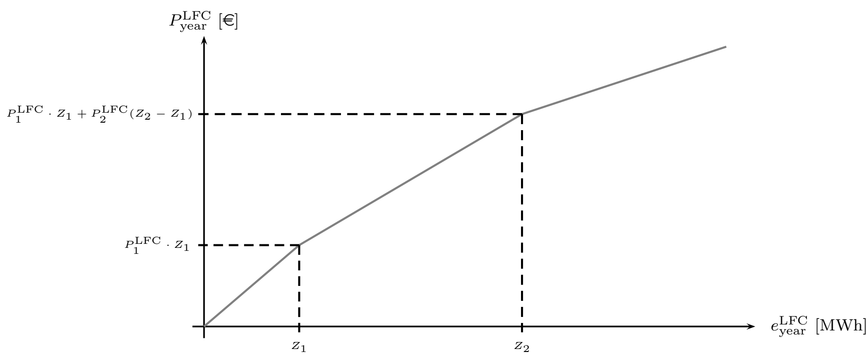

Fig. 2. Piece-wise linear price curve for the load following contract (on an annual basis)

This price system is adjusted annually. When using it on a daily basis, it leads to the following effect. At the beginning of the year, we are always in zone 1, growing steadily into zone 2 and resulting finally in zone 3 at a particular point of time. With this interpretation of the model, the effective current price depends on the relative position of the day within the year. This leads to difficulties for the short term modeling. To overcome this problem, we introduce a daily based model in Section 3.1.

The amount of energy from LFC is in principle unlimited and can vary in each of the quarter-hour time periods without restrictions. Hence, no additional constraints for the energy purchase from the load following contract are needed.

3 Mathematical Formulation

In this section, we formulate the described problem above as a MILP problem. Our task is to minimize the total cost while meeting the demand forecast for each quarter-hour time interval and while meeting the constraints associated with the power plant usage.

3.1 Objective Function

The total cost c^{tot} for the fulfillment of the demand for the particular day d consists of the cost for the power plant operation, c^{PP}, the cost for the purchase of power from the spot market, c^{SM}, and the cost for the purchase of power from the load following contract, c^{LFC}. Hence, we get for the total cost

$$c^{\mathrm{tot}} = c^{\mathrm{PP}} + c^{\mathrm{SM}} + c^{\mathrm{LFC}} \quad . \tag{6}$$

Let us now discuss the three cost components in detail.

Cost for the Power Generation in the Own Power Plant

The cost associated with the power plant is given by the sum of the fix cost $C^{\mathrm{PP}}_{\mathrm{fix}}$ and the variable cost $C^{\mathrm{PP}}_{\mathrm{var}}$ per MWh. Recognize that the variable cost represent the cost for the produced energy and the fixed cost include the electric power cost; *i.e.* the power capacity of the plant influences the construction cost of the plant which are included in the fixed cost $C^{\mathrm{PP}}_{\mathrm{fix}}$. We can then write the total cost in € as

$$c^{\mathrm{PP}} = C^{\mathrm{PP}}_{\mathrm{fix}} + C^{\mathrm{PP}}_{\mathrm{var}} \cdot e^{\mathrm{PP}} \quad , \tag{7}$$

where e^{PP} is the total energy withdrawn from the power plant. If we denote by p^{PP}_t the electric power in MW of the power plant during time slice t, then we get

$$e^{\mathrm{PP}} = \sum_{t=1}^{N^{\mathrm{T}}} \frac{1}{4} p^{\mathrm{PP}}_t \quad . \tag{8}$$

Cost for the Purchase of Energy from the Spot Market

As introduced in Section 2.2, let α be the number of base load and β be the number of peak load contracts. The electric power purchased per time interval t (quarter-hour) is then given by

$$p^{\mathrm{SM}}_t = \alpha + I^{\mathrm{PL}}_t \cdot \beta \quad , \tag{9}$$

with the usage of step function I^{PL}_t for the peak load contracts. From the description of Section 2.2, they are active within 48 quarter-hour intervals respectively 12 hours

$$I_t^{\mathrm{PL}} = \begin{cases} 0, & t = 1, \dots, 32 \quad \text{and} \quad t = 81, \dots, 96 \\ 1, & t = 33, \dots, 80 \end{cases} \quad . \tag{10}$$

The payment has to be made over the total energy amount in MWh delivered, resulting in

$$e^{\mathrm{SM}} = \sum_{t=1}^{N^{\mathrm{T}}} \frac{1}{4} p_t^{\mathrm{SM}} = \sum_{t=1}^{N^{\mathrm{T}}} \frac{1}{4} \left(\alpha + I_t^{\mathrm{PL}} \cdot \beta \right) = 24 \cdot \alpha + 12 \cdot \beta \quad . \tag{11}$$

Finally, the cost for the purchase of energy from the spot market at day d are determined by the bourse. They are C^{BL} € per MWh for the products base load and C^{PL} € per MWh for peak load, respectively. Finally, this yields to the cost

$$c^{\mathrm{SM}} = \sum_{t=1}^{N^{\mathrm{T}}} \frac{1}{4} \left(C^{\mathrm{BL}} \cdot \alpha + C^{\mathrm{PL}} \cdot I_t^{\mathrm{PL}} \cdot \beta \right) = 24 \cdot C^{\mathrm{BL}} \cdot \alpha + 12 \cdot C^{\mathrm{PL}} \cdot \beta \quad , \tag{12}$$

associated with the purchase of energy from the spot market. As the electric power for the base load and peak load contracts is constant, there is no additional cost for the electric power associated with the base load and peak load contracts.

Cost for the Energy Purchase from the Load Following Contract

In Section 2.3, we saw that the price of the LFC is given as the sum of the power rate and the variable cost per MWh purchased, the energy rate.

The power rate $C_{\mathrm{PR}}^{\mathrm{LFC}}$ is given through formula (5), which depends on the maximum yearly power level $p_{\max}^{\mathrm{LFC}}$ with respect to quarter hours. Notice that optimization could lead to the scenario that for a short time period high power is drained which contribute only very little energy but result in high energy peaks implying a high power rate. In order to avoid such situations, we introduce an electric power reference level $P_{\mathrm{ref}}^{\mathrm{LFC}}$ which is not allowed to be exceeded by the electric power purchased from the LFC. This reference level could either be the highest measured value so far, a corresponding last year value, an arbitrary limit which is not allowed to be exceeded, or a reference level determined by a long-run optimization model. Hence, we want to satisfy the following constraint

$$p_t^{\mathrm{LFC}} \leq P_{\mathrm{ref}}^{\mathrm{LFC}} \quad , \qquad \forall t \quad , \tag{13}$$

with p_t^{LFC} being the electric power from the LFC for time slice t. This hard constraint on p_t^{LFC} allows us to substitute $p_{\max}^{\mathrm{LFC}}$ in formula (5) by $P_{\mathrm{ref}}^{\mathrm{LFC}}$. Hence, the power rate reduces to fixed cost on an annual basis. As our model is a short term optimization model, these costs are not relevant. Therefore, the cost for the purchase from the load following contract is given by the energy rate $c_{\mathrm{ER}}^{\mathrm{LFC}}$, which are variable cost per MWh, as

$$c^{\mathrm{LFC}} = c^{\mathrm{LFC}}_{\mathrm{ER}} \quad . \tag{14}$$

Now, consider the special zone prices of the load following contract described in Section 2.3. As already mentioned, the annually based price system is improper for our optimization model. To overcome this difficulties, we split the zones into daily quantities and simulate daily zones. Instead of using Z_1 and Z_2, the zonal borders Z_1^{d} and Z_2^{d} are utilized with

$$Z_1^{\mathrm{d}} = Z_1/365 \quad , \quad Z_2^{\mathrm{d}} = Z_2/365 \quad . \tag{15}$$

With e^{LFC} as the daily delivery quantity from the load following contract

$$e^{\mathrm{LFC}} := \sum_{t=1}^{N^{\mathrm{T}}} \frac{1}{4} p_t^{\mathrm{LFC}} \quad , \tag{16}$$

we have that the quantity price of one day is given by

$$c^{\mathrm{LFC}} = \begin{cases} P_1^{\mathrm{LFC}} \cdot e^{\mathrm{LFC}}, & \text{if } 0 \leq e^{\mathrm{LFC}} \leq Z_1^{\mathrm{d}} \\ P_1^{\mathrm{LFC}} \cdot Z_1^{\mathrm{d}} + P_2^{\mathrm{LFC}} \left(e^{\mathrm{LFC}} - Z_1^{\mathrm{d}} \right), & \text{if } Z_1^{\mathrm{d}} < e^{\mathrm{LFC}} \leq Z_2^{\mathrm{d}} \\ P_1^{\mathrm{LFC}} \cdot Z_1^{\mathrm{d}} + P_2^{\mathrm{LFC}} \left(Z_2^{\mathrm{d}} - Z_1^{\mathrm{d}} \right) + P_3^{\mathrm{LFC}} \left(e^{\mathrm{LFC}} - Z_2^{\mathrm{d}} \right), & \text{if } Z_2^{\mathrm{d}} < e^{\mathrm{LFC}} \end{cases} \quad .$$

In order to keep the model generic, we assume to have N^{B} different zones; where $b \in \mathcal{B}$ is one of the zones; *i.e.* $b \in \mathcal{B} := \{1, \ldots, N^{\mathrm{B}}\}$. In our case we have $N^{\mathrm{B}} = 3$. To identify the appropriate prize segments, we use the binary variables μ_b. These variables indicate in which interval the daily purchased amount of energy lies, that is

$$\mu_b := \begin{cases} 1, & \text{if } Z_{b-1}^{\mathrm{d}} \leq e^{\mathrm{LFC}} < Z_b^{\mathrm{d}} \\ 0, & \text{otherwise} \end{cases} \quad , \qquad b = 1, \ldots, N^{\mathrm{B}} \quad , \tag{17}$$

where we define for notational convenience $Z_0^{\mathrm{d}} = 0$ and $Z_{N^{\mathrm{B}}}^{\mathrm{d}}$ as a number large enough. Let variable e_b^{LFC} be the contribution to e^{LFC} in segment b. Then we get that the equalities

$$\sum_{b=1}^{N^{\mathrm{B}}} \mu_b = 1 \tag{18}$$

and

$$e^{\mathrm{LFC}} = \sum_{b=1}^{N^{\mathrm{B}}} \left(Z_{b-1}^{\mathrm{d}} \mu_b + e_b^{\mathrm{LFC}} \right) \quad , \tag{19}$$

as well as the inequalities

$$e_b^{\mathrm{LFC}} \leq \left(Z_b^{\mathrm{d}} - Z_{b-1}^{\mathrm{d}} \right) \mu_b \quad , \qquad b = 1, \ldots, N^{\mathrm{B}} \tag{20}$$

connect variables e_b^{LFC} and μ_b to the energy e^{LFC} purchased from the LFC. Hence, we get for the energy rate of the load following contract

$$c_{\mathrm{ER}}^{\mathrm{LFC}} = \sum_{b=1}^{N^{\mathrm{B}}} \left(C_b^{\mathrm{LFC}} \cdot \mu_b + P_b^{\mathrm{LFC}} \cdot e_b^{\mathrm{LFC}} \right) \quad , \tag{21}$$

where C_b^{LFC} are the accumulated cost up to segment b, *i.e.*,

$$C_b^{\mathrm{LFC}} = \begin{cases} 0, & \text{if } b = 1 \\ P_1^{\mathrm{LFC}} \cdot Z_1^{\mathrm{d}}, & \text{if } b = 2 \\ C_{b-1}^{\mathrm{LFC}} + P_{b-1}^{\mathrm{LFC}} \left(Z_{b-1}^{\mathrm{d}} - Z_{b-2}^{\mathrm{d}} \right), & \text{if } b = 3, \ldots, N^{\mathrm{B}} \end{cases} \tag{22}$$

The breaking down of the zone prizes on a daily basis is a trick to present the special price structure of the LFC. In practice, one could use the data of previous years to estimate the cost of the LCF for each day. However, such a method requires a huge amount of experience in order to adjust the price in a meaningful way and it has to be seen in practice if it would outperform the special modeling of the zone prices discussed above.

The set of variables $\mu_1, \ldots, \mu_{N^{\mathrm{B}}}$ form a so-called *Special Order Set of type 1* (SOS-1), as only one variable of the set can have a nonzero value. The SOS-1 was introduced by Beale and Tomlin in 1969 [4]. Description of SOS-1 in the context of integer programming can be found, for instance, in [21, Chapter 6.7] and [22].

3.2 Demand and Power Plant Constraints

Let us now discuss the demand constraints and the constraints for the power plant operation.

Power Demand Constraints

Clearly, we have to meet the electric power demand for each quarter-hour. That gives us

$$p_t^{\mathrm{PP}} + p_t^{\mathrm{SM}} + p_t^{\mathrm{LFC}} = P_t \quad , \qquad t = 1, \ldots, N^{\mathrm{T}} \quad . \tag{23}$$

Recognize that the power demand has to be met exactly. The reason is that (at least a large amount of) energy cannot be stored.

Power Plant Constraints

We have to discuss the modeling of the restricted operation of the power plant. Therefore, we introduce the binary variables

$$\delta_{mt} := \begin{cases} 1, \text{ if the power plant is at time } t \text{ at stage } m \\ 0, \text{ otherwise} \end{cases} \tag{24}$$

to model the stages, $m \in \mathcal{M} := \{1, \ldots, N^{\mathrm{M}} = 8\}$, of the plant. Stage $m = 1$ corresponds to the idle state of the power plant. Values $m = 2, \ldots, N^{\mathrm{M}} = 8$ refer to the capacity utilizations 0.4, 0.5, 0.6, 0.7, 0.8, 0.9 and 1, respectively. The plant is in exactly one of those stages at any time, that is

$$\sum_{m=1}^{N^{\mathrm{M}}} \delta_{mt} = 1 \quad , \qquad \forall t \quad . \tag{25}$$

The utilized power can then be calculated according to the following formula

$$p_t^{\mathrm{PP}} = P_{\mathrm{max}}^{\mathrm{PP}} \cdot \sum_{m=2}^{N^{\mathrm{M}}} \frac{1}{10} (m+2)\, \delta_{mt} \quad , \qquad \forall t \quad , \tag{26}$$

where $P_{\mathrm{max}}^{\mathrm{PP}}$ is the capacity of the power plant in MW. Note that this is the counter part of equation (2) with binary variables but holds also true when the plant is in the idle stage 1.

In equation (3), we formulated the requirement that any power stage has to be continued for at least two hours. This constraint is called *minimum up time constraint*. For this purpose, the binary variables χ_t^{S} keep track, if there is a change in the power plant level in time slice t

$$\chi_t^{\mathrm{S}} \geq \delta_{mt} - \delta_{mt-1} \quad , \qquad \forall m \quad , \quad t = 2, \ldots, N^{\mathrm{T}} \quad , \tag{27}$$

and

$$\chi_t^{\mathrm{S}} \geq \delta_{mt-1} - \delta_{mt} \quad , \qquad \forall m \quad , \quad t = 2, \ldots, N^{\mathrm{T}} \quad . \tag{28}$$

Inequalities (27) and (28) ensure that variable χ_t^{S} has value 1, if there is a change in the stage of the plant; however, χ_t^{S} can also have value 1, if there was no change in the stage. It is only important that it is now possible to formulate the condition

$$\chi_t^{\mathrm{S}} + \chi_{t+1}^{\mathrm{S}} + \chi_{t+2}^{\mathrm{S}} + \chi_{t+3}^{\mathrm{S}} + \chi_{t+4}^{\mathrm{S}} + \chi_{t+5}^{\mathrm{S}} + \chi_{t+5}^{\mathrm{S}} + \chi_{t+6}^{\mathrm{S}} + \chi_{t+7}^{\mathrm{S}} \leq 1 \quad , \\ t = 1, \ldots, N^{\mathrm{T}} - 7$$

or generally

$$\sum_{k=1}^{D_{\mathrm{act}}^{\mathrm{PP}}} \chi_{t+k-1}^{\mathrm{S}} \leq 1 \quad , \qquad t = 1, \ldots, N^{\mathrm{T}} - \left(D_{\mathrm{act}}^{\mathrm{PP}} - 1\right) \quad , \tag{29}$$

ensuring that within any two hours, or $D_{\mathrm{act}}^{\mathrm{PP}} = 8$ time intervals, at most one stage change takes place.

In addition to the restrictions above, we discussed in Section 2.1 also the requirement for any idle period to be at least four hours. This condition is called *minimum idle time requirement* or *minimum down time requirement*. Let us introduce the binary variable χ_t^{I}, indicating if the power plant has been started, *i.e.* if it left the idle state in time slice t. We get the following inequalities

$$\chi_t^{\mathrm{I}} \geq \delta_{1t-1} - \delta_{1t} \quad , \qquad t = 2, \ldots, N^{\mathrm{T}} \quad . \tag{30}$$

The condition for the idle period given in (4), can then be modeled as

$$\sum_{k=1}^{D_{\mathrm{idl}}^{\mathrm{PP}}} \chi_{t+k-1}^{\mathrm{I}} \leq 1 \quad , \qquad t = 1, \ldots, N^{\mathrm{T}} - \left(D_{\mathrm{idl}}^{\mathrm{PP}} - 1\right) \tag{31}$$

with $D_{\mathrm{idl}}^{\mathrm{PP}} = 16$, or four hours respectively. Constraint (31) can be interpreted in the way that the power plant is not allowed to leave the idle state more than once within any $D_{\mathrm{idl}}^{\mathrm{PP}}$ time slices.

As already mentioned in Section 2.1, we relax the condition of the minimum up and idle time for the beginning and the end of the planning horizon. However, for $t = 1$, $t = N^{\mathrm{T}} - \left(D_{\mathrm{act}}^{\mathrm{PP}} - 1\right)$, we have that the stage of the power plant is allowed to change only once in the first, last, $D_{\mathrm{act}}^{\mathrm{PP}}$ time slices.

The variables χ_t^{S} are initially binary variables indicating a change of the stage of the power plant. However, we can relax these variables to be non-negative continuous. The reason is that constraints (27), (28) and (29) force the variables χ_t^{S} to be binary in the case that the minimum uptime condition is tight, as the right-hand-side of constraints (27), (28) and (30) can only take the values 0 and 1. Recognize that this does not mean that the left hand side of constraints (29) being equal to 1 implies that the variables χ_t^{S} are binary. From the modeling point of view, it is therefore equivalent to use a binary or a non-negative continuous domain for variables χ_t^{S}. However, computationally, there is a difference[5]. The reason is that most Branch & Bound and Branch & Cut algorithms use LP domain relaxations, treating binary variables as continuous [44, 2]. The branching process ensures then that those continuous variables are forced to be integral. In case of variable χ_t^{S}, we do not want the solver to branch on those, as their integrality is already applied by the binary variables δ_{mt}. However, if we can "forbid" the solver to branch on those variables (in GAMS this is accomplished by setting the priorities to `+inf`), then these two approaches of modeling the domain are also computationally equivalent[6]. The same concept holds also true for the variables χ_t^{I}.

This idea of avoiding to branch on variables χ_t^{S} and χ_t^{I} can be realized in the modeling language GAMS by defining branching priorities for these

[5] For the real data of Stadtwerke Saarlouis, the running time of the continuous model compared to the binary model was less than 40%, it needed 45% of the iterations and 60% of the branching nodes.

[6] Recognize that for this argument to be correct, we need also that the heuristics treat both the binary and the continuous case equivalently as well as factional solutions for the variables χ_t^{S} and χ_t^{I} are not rejected by the heuristics and during the branching process. However, just setting the branching priorities low, *i.e.* to value 10, has already a significant impact. For our case of the real data, the running time decreased by 30%.

variables, [31, 32, 6]. The default branching priority for integral variables in GAMS is value 1. The higher the value, the lower is the priority to branch on these variables. The GAMS code for our case can then look as follows

```
*
* avoid branching on variables "chiS(t)" and "chiI(t)"
*
   chiS.prior(t) = +inf;
   chiI.prior(t) = +inf ;

* use the branching priorities in the model
   portfolio.prioropt = 1 ;
```

Defining an arbitrary value > 1 for the branching priority for the variables χ_t^{S} and χ_t^{I} ensures that the branching on those variables is done only after all other variables have integral value. However, as the integrality of the variables δ_{mt} does not imply the variables χ_t^{S} and χ_t^{I} to be binary, it might be needed to branch on those variables nevertheless. One way where such a branching is not necessary is the case when there is a (non-zero) cost associated with the variables χ_t^{S} and χ_t^{I}; for instance start-up cost for the power plan, see Section 4.2.

Carrion and Arroyo give a compact formulation of the minimum up and minimum idle time constraints using only one set of binary constraints – instead of two sets of variables χ_t^{S} and χ_t^{I} [9]. However, they have a quadratic cost structure for the power plant and binary variables indicating if the power plant is used or not. Gröwe-Kuska et al. [18] also use binary variables indicating if the plant is used in time slice t or not. Hence, they can also model the minimum up/down time requirement without using additional binary variables.

4 Improvements of the Model Formulation

4.1 Assumptions and Limitations of the Model

Here, we discuss the assumptions needed for our model and present some limitations.

1. The pricing for the load following contract is very simplified. In practice, there are special rebates; *e.g.* they depend on the total energy purchased or the ratio of energy purchased to maximal power drained.
2. Although the electric power forecast is accurate enough for about a week, the increase of the time horizon to two or more days is computationally expensive and thus limits the application of this model.
3. As public services in Germany usually do not sell energy in the spot market, our model does not include this feature. Indeed, allowing to trade excess energy, leads to a different kind of optimization problem: One would operate the own power plant at an optimal efficient level and optimize the sell and purchase of the remaining / excess energy in the market.

An overview of the behavior of such a market can be found in the book edited by Schweppe et al. [33].

4.2 Modifications

- **EEG: Renewable Energy Act:** A law to regulate the priority of renewable energies in Germany; last change on June 14, 2006 [7, 8]. Especially the expansion of wind energy is intended. It forces electric distributor having wind-energy plants in their portfolio for their service area. Hence, it forces the additional purchase of wind-energy. However, the exact amount produced by wind is unpredictable. The optimization model has to treat this energy source stochastically. Stochastic optimization models and algorithms for this topic have been widely discussed in literature.
- **Hour Contracts:** The power bourse EEX also offers hour contracts which refer only to a specific hour. Those hour contracts can be used to fill up some small portion of the portfolio which is not covered by the base load and peak load contracts.
- **Emission Modeling:** The environmental issues in power generation play an important role. Especially the emissions of CO_2, NO_x or SO_x are currently under restriction. This can be modeled, for instance, via hard or soft constraints on the generated emissions or by minimizing the cost associated with those emission. However, in the latter case, it is difficult to derive appropriate costs for the emissions. This problem is called *environmental dispatch problem*. More details can be found, for instance, in [40, 46].
- **Efficiency Factor under Partial Load:** The efficiency factor of a power plant decreases when it is operated only under partial load. In particular, the variable costs are not constant through the whole power range. Hence, for each power stage, a separate cost has to be assumed. This is not so much a problem from the point of view of the mathematical modeling, but it is particularly difficult to get realistic data; *i.e.* the cost coefficients.

  Let $C_{\mathrm{m}}^{\mathrm{PP}}$ be the variable cost in € per MWh for the power plant when operated in stage $m \in \mathcal{M}$, $m \geq 2$. If those data are available, then we can substitute the variable cost $C_{\mathrm{var}}^{\mathrm{PP}} \cdot e^{\mathrm{PP}}$ of the power plant in equation (7) by

  $$\frac{1}{40} P_{\max}^{\mathrm{PP}} \sum_{t=1}^{N^{\mathrm{T}}} \sum_{m=2}^{N^{\mathrm{M}}} C_{\mathrm{m}}^{\mathrm{PP}} \left(m+2\right) \delta_{mt} \quad .$$

  Recognize that we do not need any additional variables or constraints.
- **Start-up Cost for the Power Plant:** In equation (7), we stated that the cost of the power plant consists of fixed cost $C_{\mathrm{fix}}^{\mathrm{PP}}$ and variable cost $C_{\mathrm{var}}^{\mathrm{PP}}$ per MWh produced by the plant. Those fixed cost apply whether we use the power plant during this day or not. Such fixed cost can be for instance capital cost. However, it is more realistic, to have also start-up

cost which occur whenever the power plant is operated from an idle state. Those cost are typically fuel-costs for warming up.

Let $C_{\mathrm{su}}^{\mathrm{PP}}$ be the start-up cost for the power plant. Then, we can add the following cost

$$C_{\mathrm{su}}^{\mathrm{PP}} \sum_{t=1}^{N^{\mathrm{T}}} \chi_t^{\mathrm{I}}$$

to the cost of the power plant c^{PP} given in equation (7).

Similarly, one could define shut-down cost for the plant. However, in this case, additional variables would be needed. Recognize that we can also include stage switching cost, applying whenever the power plant changes its stage of operation.

- **Down-Time or Forced Operation of the Power Plant:** In practice, it could occur that the power plant has to be shut-down for some time period; *e.g.* due to scheduled maintenance. This can be handled straight forward with our model by defining

$$\delta_{1t} = 1 \quad ,$$

for all time slices t where we want to force the plant to be in idle state. This condition implies for a given t that $\delta_{mt} = 0$ for all $m \in \mathcal{M}$, $m \geq 2$ according to constraint (25).
This can be easily done in GAMS with the following code

```
*
* force the power plant to be shut-down in time slice 't17'
* i.e. to be in idle state in time slice 't17'
*
 delta.fx('m1','t17') = 1 ;
```

The same idea can be used to force the power plant to operate in a certain stage $m \in \mathcal{M}$, $m \geq 2$ or just not to be in the idle stage. Recognize that in all cases, the number of binary variables in our model are reduced.

5 Computational Results

The optimization model is implemented in GAMS, version 22.7. The code is included in the GAMS model library [16] with the model name `poutil.gms`. All computations are done with a Pentium Intel Centrino Dual 2.00 GHz with 1 GB RAM and Windows XP platform. In order to achieve computational results which are comparable, we use only one processor. We observed that with two processors, the speed-up time is almost linear in average.

A GAMS code to use multiple processors looks as follows

```
*
* for parallel use of cplex
*
* create file ‘cplex.opt’
* and set the number of threads to 2
 $ onecho > cplex.opt
    threads 2
 $ offecho

* use the option file ‘cplex.opt’ for the ‘energy’ model
 energy.optfile = 1 ;
```

Using the real data for the year 2003 for the Stadtwerke Saarlouis [37], a German distributor, we get a (proven) optimal solution within 987 seconds. The computational details are given in the first row of Table 1 and the solution is plotted in Figure 3. The total energy demanded is given in the area below the power demand forecast.

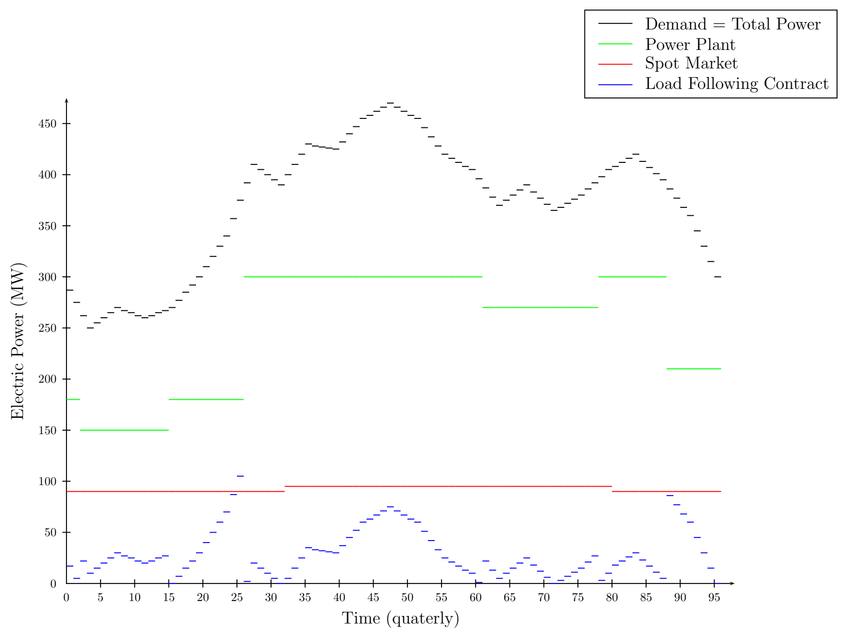

Fig. 3. Optimal solution for real data of Stadtwerke Saarlouis

Table 1 shows computational results for different electric power demand forecasts. The basis are some real data for the power forecast. The new power forecast are randomly generated within a 2% tolerance. The column with label “# Nodes” gives the number of nodes in the Branch & Bound tree. The running time is stated in the last column and is measured in seconds. In all 10 cases, the energy purchased from the LFC was enough to be in the cheapest price segment three. The borderline from price segment two to three is 500 MWh on a daily basis. Interestingly, the solutions differ quite remarkable when the energy forecast changes slightly; especially the purchase of energy from the spot market differ a lot.

Table 1. Computational results for different demand forecast. The first row are the real data and all other data are (uniform) randomly generated within an absolute difference of 2%

#	Power Plant		Spot Market				LFC		c	# Nodes	CPU
	e^{PP}	c^{PP}	α	β	e^{SM}	c^{SM}	e^{LFC}	c^{LFC}			
1	6,015.0	150,375.0	90	5	2,220	71,580	694.00	44,838	266,793.0	59,300	986.61
2	6,120.0	153,000.0	82	14	2,136	69,864	663.75	43,265	266,129.0	24,100	734.63
3	6,172.5	154,312.5	78	12	2,016	65,808	747.75	47,633	267,753.5	100,500	1678.67
4	6,045.0	151,125.0	90	0	2,160	69,120	728.25	46,619	266,864.0	50,000	992.30
5	6,142.5	153,562.5	82	10	2,088	67,896	726.00	46,502	267,960.5	53,800	1511.08
6	6,165.0	154,125.0	80	11	2,052	66,852	723.75	46,385	267,362.0	87,100	1225.34
7	5,977.5	149,437.5	94	0	2,256	72,192	713.75	45,865	267,494.5	53,300	1550.58
8	6,292.5	157,312.5	71	18	1,920	63,384	707.25	45,527	266,223.5	41,700	1020.03
9	6,202.5	155,062.5	79	11	2,028	66,084	714.75	45,917	267,063.5	48,400	2020.84
10	6,202.5	155,062.5	79	9	2,004	65,100	727.75	46,593	266,755.5	51,400	845.41

In Table 2, the computational results for different minimum duration times between state changes of the power plant are shown. The power forecast are the same in all computations. We can observe that the change in the duration does not effect the solution very much. In fact, the difference in the total cost between a duration time of 1 hour and 4 hours is less than 2%. One explanation can be found in Figure 3 as the power level of the power plant does not change every 2 hours. Hence, a change in the duration has not such a big effect. As expected, the computational running time decreases when increasing the duration D_{act}^{PP}. An optimal solution for the duration of 4 hours is shown in Figure 4.

Table 2. Computational results for different minimum duration times D_{act}^{PP} between state changes of the power plant

D_{act}^{PP}	Power Plant		Spot Market				LFC		c	# Nodes	CPU
	e^{PP}	c^{PP}	α	β	e^{SM}	c^{SM}	e^{LFC}	c^{LFC}			
4	6,112.5	152,812.5	90	5	2,220	71,580	596.50	39,768	264,160.5	1471,000	15039.56
6	6,075.0	151,875.0	90	5	2,220	71,580	634.00	41,718	265,173.0	165,700	1736.77
8	6,015.0	150,375.0	90	5	2,220	71,580	694.00	44,838	266,793.0	59,300	986.61
10	6,022.5	150,562.5	90	2	2,184	70,104	722.50	46,320	266,986.5	25,300	685.06
12	6,165.0	154,125.0	75	17	2,004	65,964	760.00	48,270	268,359.0	19,500	798.44
14	6,060.0	151,500.0	80	15	2,100	68,820	769.00	48,738	269,058.0	15,900	459.86
16	6,060.0	151,500.0	80	15	2,100	68,820	769.00	48,738	269,058.0	7,700	358.73

We also made some computational tests for the case of a two-day planning horizon, $N^{\mathrm{T}} = 192$. The tested instance could not be solved to global optimality and after 10 hours of computation time, the gap was still 5.99%.

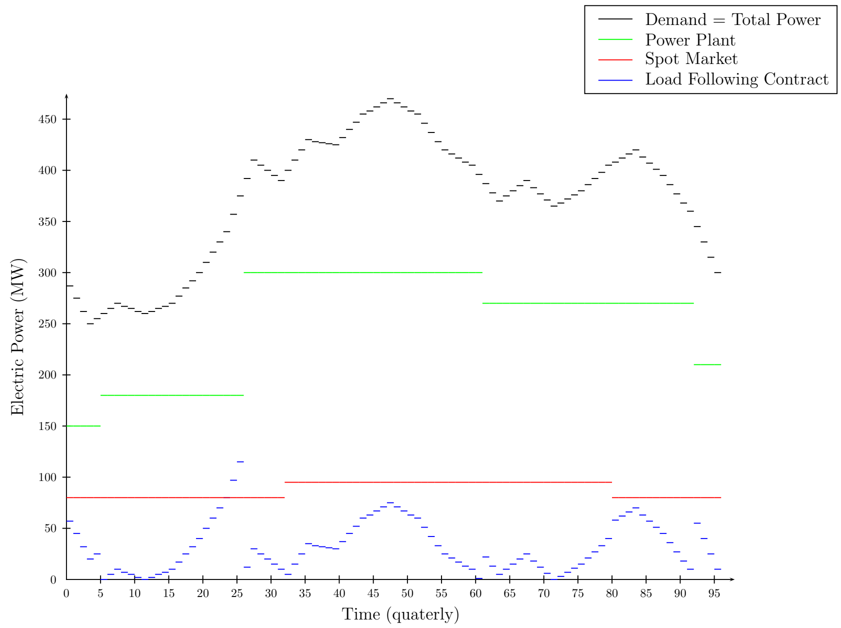

Fig. 4. Optimal solution for real data of Stadtwerke Saarlouis with $D_{act}^{\mathrm{PP}} = 16$ (4 hours)

6 Conclusion

In this article, we developed a model for the portfolio optimization of an electric services distributor. This study was motivated by a real case of Germany public services. It brings together the real energy world and mathematical optimization. The model is very generic and can be easily extended with additional features but nevertheless, it has an appropriate degree of details matching the real world case. We also showed that the developed model is computationally effective for one-day ahead planning. The developed model has also didactic value as some modeling tricks and their computational implications are discussed. The GAMS code is available in the GAMS model library [16].

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A Indices and Index Sets

The indices, index sets and the indicator function of the mathematical programming model of Section 3 are given in the first column of Table 3. The second column states the name of the corresponding set / function used in the GAMS model `poutil.gms` included in the GAMS model library [16]. The third column gives some explanations along with the size of the sets.

The model is generic and can tolerate in principle any number of time slices `Nt`. However, when changing the planning horizon, the modeling of the spot market has to be adjusted; *e.g.* there has to be a variable α and β for each day of the planning horizon. In addition, the zones for the LFC have to be adjusted for the new horizon; *e.g.* the step function I_t^{PL} in formula (10) has to be redefined.

In principle, the model can handle any number of power plant stages `Nm`. However, when changing this number, the formula for the power level p_t^{PP}, stated in equation (26), has to be changed, too.

Table 3: Indices, index sets and indicator function

$t \in \mathcal{T} := \{1, \ldots, N^{\mathrm{T}}\}$	`t`	Set of time slices per day. The day is split in N^{T} time intervals of 15 minutes each. $N^{\mathrm{T}} = 96$
$m \in \mathcal{M} := \{1, \ldots, N^{\mathrm{M}}\}$	`m`	Set for the power level stages of the power plant. The first stage corresponds to the stationary or idle phase of the plant; all other stages correspond to the 60% – 100% plant utilization stages. $N^{\mathrm{M}} = 8$
$b \in \mathcal{B} := \{1, \ldots, N^{\mathrm{B}}\}$	`b`	Set of support points of the zone prices for the LFC. $N^{\mathrm{B}} = 3$
I_t^{PL}	`IPL(t)`	Indicator function for the peak load contract. It is defined in equation (10)

B Variables

All variables used in the mathematical model are summarized in the first column of Table 4. The corresponding variable name of the GAMS model `poutil.gms`, included in the GAMS model library [16], is given in the second column. A "-" in the second column states that this variable is not used in the GAMS model formulation, *e.g.* the variable could be substituted by other variables. The units are stated in []-brackets in the third column and the forth column gives the type of the variable in the GAMS model formulation. $\mathbb{R}_+$, $\mathbb{Z}_+$, $\{0, 1\}$ means that the variable is non-negative continuous, non-negative integer or binary, respectively. Recognize that this does not represent the domain of the variable but the type of the variable in the GAMS model. Particularly, the binary variables χ_t^{S} and χ_t^{I} are modeled being non-negative continuous; see Section 3.2.

Table 4: Variables with corresponding GAMS name, unit, model domain, equation reference(s) and explanations

Objective Function					
c^{tot}	`c`	[€]	$\mathbb{R}_+$	(6)	Total cost
Power Plant					
c^{PP}	`cPP`	[€]	$\mathbb{R}_+$	(7)	Cost associated with the power plant usage

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Table 4 – Continued from previous page

e^{PP}	-	[MWh]	$\mathbb{R}_+$	(8)	Total amount of energy withdrawn from the power plant
p_t^{PP}	`pPP(t)`	[MW]	$\mathbb{R}_+$	(26)	Amount of power withdrawn from the power plant for time slice t. This variables can only have the discrete values $0, 0.6, 0.7, 0.8, 0.9$ and 1.0 referred to the power plant capacity $P_{\max}^{\mathrm{PP}}$
δ_{mt}	`delta(m,t)`	$[-]$	$\{0,1\}$	(24)	Binary variable with value 1 if the power plant is in time interval t in stage m and 0 otherwise
χ_t^{S}	`chiS(t)`	$[-]$	$\mathbb{R}_+$	(27), (28), (29)	Binary variable with value 1 if the power plant changes its stage at the beginning of time interval t and 0 otherwise
χ_t^{I}	`chiI(t)`	$[-]$	$\mathbb{R}_+$	(30)	Binary variable with value 1 if the power plant has been started up at the beginning of time interval t and 0 otherwise; *i.e.* the power plant left the idle condition
Spot Market					
c^{SM}	`cSM`	[€]	$\mathbb{R}_+$	(12)	Cost for the energy purchase from the spot market
e^{SM}	-	[MWh]	$\mathbb{R}_+$	(11)	Energy purchased from the spot market
p_t^{SM}	`pSM(t)`	[MW]	$\mathbb{R}_+$	(9)	Electric power from the spot market for time slice t resulting from base load and peak load contracts
α	`alpha`	$[-]$	$\mathbb{Z}_+$		Quantity / proportion of the base load contracts of the portfolio contribution bought from the spot market. Typical range is between 0 and 200. We set as an upper bound the maximal demand in the planning horizon
β	`beta`	$[-]$	$\mathbb{Z}_+$		Quantity / proportion of the peak load contracts of the portfolio contribution bought from the spot market. Typical range is between 0 and 200. We set as an upper bound the maximal demand in the planning horizon
Load Following Contract					
c^{LFC}	`cLFC`	[€]	$\mathbb{R}_+$	(14)	Cost for the energy purchase from load following contract: energy rate
e^{LFC}	`eLFCtot`	[MWh]	$\mathbb{R}_+$	(16)	Total energy from the load following contract

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Table 4 – Continued from previous page

e_b^{LFC}	`eLFCs(b)`	[MWh]	$\mathbb{R}_+$	(20)	Contribution to the total energy of the LFC in segment b
p_t^{LFC}	`pLFC(t)`	[MW]	$\mathbb{R}_+$	(13)	Power from the load following contract for time slice t
μ_b	`mu(b)`	[−]	$\{0,1\}$	(17)	Binary variables with value 1 if the daily purchased amount of energy lies between Z_{b-1}^{d} and Z_b^{d}

C Constraints

All constraints of the GAMS model `poutil.gms`, included in the GAMS model library [16], are summarized in the Table 5. The first column of Table 5 states the name of the constraint in the GAMS model, the second column gives the corresponding equation number of the mathematical programming formulation introduced in Section 3 and the third column gives a brief explanation.

Table 5: Constraints of the GAMS model with corresponding equation number and explanations

Objective Function		
`obj`	(6)	Total cost
Power Demand		
`demand(t)`	(23)	Power demand constraint for each time slice t (quarter-hour)
Power Plant		
`PPcost`	(7)	Power plant cost. The fixed cost of the power plant are not included in this model
`PPpower(t)`	(26)	Power of power plant at time slice t
`PPstage(t)`	(25)	The power plant is in exactly one stage at any time slice t
`PPchiS1(t,m)`	(27)	Constraint on variable `chiS(t)` to track a stage change m at time slice t
`PPchiS2(t,m)`	(28)	Constraint on variable `chiS(t)` to track a stage change m at time slice t

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Table 5 – Continued from previous page

`PPstageChange(t)`	(29)	At most one stage change takes place within any $D^{\mathrm{PP}}_{\mathrm{act}}$ time slices
`PPstarted(t)`	(30)	Constraint on variable `chiI(t)` to indicate if the plant left the idle state at the beginning of time slice t
`PPidleTime(t)`	(31)	The idle time of the power plant has to last for at least $D^{\mathrm{PP}}_{\mathrm{idl}}$ time slices
Spot Market		
`SMcost`	(12)	Cost for the power from the spot market
`SMpower`	(9)	Power from the spot market
Load Following Contract		
`LFCcost`	(21)	Cost for the power from the LFC as the energy rate
`LFCenergy`	(16)	Energy from the LFC for one day via LFC power
`LFCmu`	(18)	Constraint on the price segment
`LFCenergyS`	(19)	Energy from the LFC for one day via energy from the different segments
`LFCemuo`	(20)	Accumulated energy amount for the first segment
`LFCemug(b)`	(20)	Accumulated energy amount for all segments except the first one

D Input Data and Parameters

All input data / parameters of the mathematical model are stated in the first column of Table 6. The corresponding name of the GAMS model `poutil.gms`, included in the GAMS model library [16], is given in the second column; a "-" states that this variable is not used in the GAMS model formulation. The particular units of the data are given the []-brackets in the third column. Column four states some explanations as well as the value of the parameters. One instance, defining the values of the data, is given in the GAMS model `poutil.gms`.

The fixed cost $C^{\mathrm{PP}}_{\mathrm{fix}}$ of the power plant are not included in the model as they are irrelevant for the optimization decisions.

Table 6: Input data / parameters with corresponding GAMS name, unit and explanations

Power Demand			

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Table 6 – Continued from previous page

P_t	`PowerForecast(t)`	[MW]	Power demand forecast on a quarter-hour base
Power Plant			
$C_{\text{fix}}^{\text{PP}}$	–	[€]	Fix cost of the power plant
$C_{\text{var}}^{\text{PP}}$	`cPPvar`	[€/MWh]	Variable cost of the power plant
$P_{\text{max}}^{\text{PP}}$	`pPPMax`	[MW]	Power plant capacity in Megawatt
D_{act}^{PP}	–	[-]	Minimum number of time intervals between two consecutive stage changes of the plant. $D_{act}^{\text{PP}} = 8$. This is modeled in the GAMS code via the set `iS`
D_{idl}^{PP}	–	[-]	Minimum number of time intervals for the plant to remain in an idle period. $D_{idl}^{\text{PP}} = 16$. This is modeled in the GAMS code via the set `iI`
Spot Market			
C^{BL}	`cBL`	[€/MWh]	Cost per base load contract purchased
C^{PL}	`cPL`	[€/MWh]	Cost per peak load contract purchased
Load Following Contract			
$C_{\text{PR}}^{\text{LFC}}$	–	[€/MW]	Cost for power rate; given in formula (5)
$C_{\text{PR,year}}^{\text{LFC}}$	–	[€/MWh]	Cost for power rate on an annual basis
$P_{\text{ref}}^{\text{LFC}}$	`pLFCref`	[MWh]	Electric power reference level for load following contract
Z_b	`eLFCbY(b)`	[MWh]	Annual borders of quantity zones for LFC
Z_b^{d}	`eLFCb(b)`	[MWh]	Daily borders of quantity zones for LFC; $b \in \mathcal{B}$ and $Z_0^{\text{d}} = 0$; calculated via formula (15)
P_b^{LFC}	`cLFCvar(b)`	[€/MWh]	Variable cost/price of LFC in segment b
C_b^{LFC}	`cLFCs(b)`	[€]	Accumulated variable cost of LFC up to segment b; calculated through equation (22)